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Models and Methods for the Design and Support of Liner Shipping Networks

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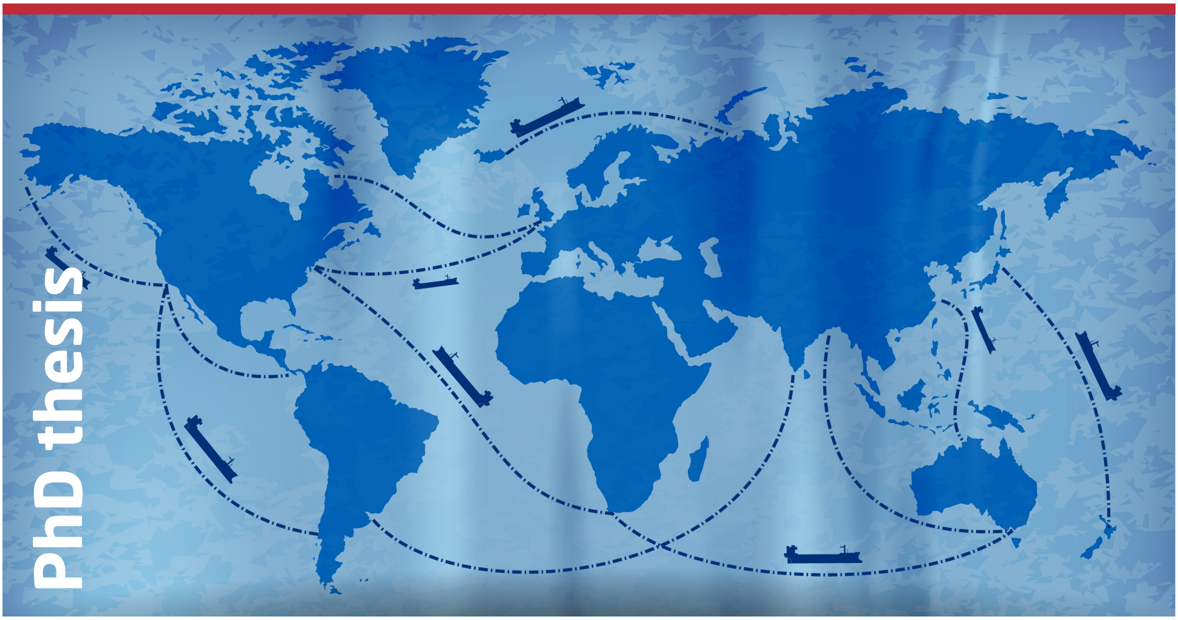

David Franz Koza
November 2017

DTU Management Engineering
Department of Management Engineering

Models and Methods for the Design and Support of Liner Shipping Networks

David Franz Koza

PhD Thesis
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Summary

The modern economy relies on cheap and reliable transportation of goods all around the globe. Liner shipping networks represent an important link in the global supply chain, as they connect countries and continents over long distances at comparatively low transportation costs. In large liner shipping networks, several hundred container vessels operate more than a hundred shipping routes. The individual routes are linked through ports, where containers can be loaded, unloaded but also be transshipped between shipping routes. The resulting networks constitute inherently complex systems.

In this thesis we present mathematical modeling and optimization tools that help decision makers in the liner shipping industry to find solutions to complex decision problems. The decision problems we address involve questions like: Which ports shall be covered by the network? How shall each single shipping route be designed to achieve a well-connected but cost-efficient network? How shall port calls be scheduled and synchronized between shipping routes to offer the most economical and fastest transportation between ports? On which route shall containers be transported, if multiple options exist?

The articles in this thesis contribute to the field of Operations Research with application in maritime optimization. More particularly, the first two articles present models and solution methods to (re-)design and (re-)schedule large liner shipping networks. The articles combine and substantially extend modeling features of previous contributions and narrow the gap to the economic and operational reality of liner shipping. The results obtained from solving the models shed light on previously unexamined issues. The developed solution algorithms cannot only handle the increased complexity inherent to the models, but improve over existing methods proposed in the literature. The third article addresses a strategic infrastructure and tanker fleet sizing problem as part of an industrial case study with a large liner shipping company. The case study is motivated by recent changes in environmental regulations that may substantially change the way liner vessels are operated in the future. The article addresses the establishment of a large-scale liquefied natural gas supply chain along a major trade lane. It analyzes the interaction between long-term investment and operational costs, derives basic decision rules and evaluates the robustness of the solutions.

Resumé (Danish)

Den moderne økonomi er afhængig af billig og pålidelig godstransport over hele kloden. Transport ved hjælp af containerskibe udgør et vigtigt element i den globale forsyningskæde. Containerskibe forbinder lande og kontinenter over lange afstande med forholdsvis lave transportomkostninger. Store containerskibsrederier (linierederier) kontrollerer flere hundrede containerskibe der typisk sejler på faster ruter. En enkelt rute deles typisk mellem adskillige containerskibe så hver havn på ruten modtager et besøg af et containerskib fra den pågældende rute en gang om ugen. De enkelte ruter er forbundet via havne, hvor containere kan lastes, losses, men også omlastes mellem skibe. Kombinationen af containerskibsruter og havne danner et kompliceret transportnetværk der muliggør transport af containere mellem de fleste større havne i verden.

I denne afhandling præsenterer vi matematiske modellerings- og optimeringsværktøjer, der hjælper beslutningstagere inden for containerskibssejlads til at finde løsninger på komplekse beslutningsproblemer. Beslutningsproblemerne vi adresserer indebærer spørgsmål som: Hvilke havne skal være omfattet af transportnetværket? Hvordan skal hver rute udformes for at opnå et godt forbundet og omkostningseffektivt netværk? Hvordan skal havnebesøg planlægges og synkroniseres mellem ruterne for at give den mest økonomiske og hurtigste transport? På hvilken rute skal containere transporteres, hvis der findes flere muligheder? Artiklerne i denne afhandling bidrager til forskning indenfor operationsanalyse med anvendelse i maritim optimering. Nærmere bestemt fremlægger de to første artikler modeller og løsningsmetoder til design og re-planlægning af store netværk indenfor containerskibsfart.

Artiklerne kombinerer og udvider modelleringsegenskaberne væsentligt i forhold til tidligere bidrag og mindsker afstanden mellem akademiske prototyper og de reelle problemstillinger som linierederier står overfor. Resultaterne, opnået ved at løse modellerne, kaster lys over tidligere uudforskede problemer. De udviklede løsningsalgoritmer kan ikke kun håndtere den øgede kompleksitet i modellerne, men også forbedre løsningskvaliteten sammenlignet med eksisterende metoder foreslået i litteraturen. Den tredje artikel omhandler en problemstilling indenfor dimensionering af infrastruktur og tankerflåde som et led i en industriel casestudie med et stort linjerederi. Casestudiet er motiveret af de seneste ændringer i miljøbestemmelser, som i væsentlig grad kan ændre forudsætningerne for containerskibsfart i fremtiden. Artiklen omhandler oprettelsen af en forsyningskæde der leverer flydende naturgas til en række havne langs en vigtig rute indenfor containerskibsfart. Artiklen analyserer samspillet mellem langsigtede investeringer og

driftsomkostninger. Simple beslutningsregler uddrages i artiklen og det undersøges hvor robuste de foreslåede forsyningskæder er i forhold til ændringer i forudsætningerne for modellen.

Preface

This thesis has been submitted to the Department of Management Engineering, Technical University of Denmark (DTU) in partial fulfillment of the requirements for acquiring the degree of Philosophiae Doctor (PhD). The work has been conducted between September 2014 and November 2017 under supervision of Professor Stefan Røpke and Professor David Pisinger. A visit of Professor Guy Desaulniers at the *Groupe d'études et de recherche en analyse des décisions (GERAD)* in Montréal, Canada, between February and July 2016 was part of the PhD studies.

The PhD project is part of the GREENSHIP project, a collaboration between the Technical University of Denmark, IT University of Copenhagen and Maersk Line. GREENSHIP is a project to develop tools for optimizing containerized liner shipping through improved network design and better cargo mix. The project has been funded by the Danish Council for Strategic Research and The Danish Energy Technology Development and Demonstration Program (EUDP).

The thesis consists of an introduction and four chapters addressing optimization problems in liner shipping. Three out of the four chapters are self-contained articles, of which one is published and two are submitted to peer-reviewed international journals. Two papers are co-authored and one paper is single-authored.

Kgs. Lyngby, Denmark, November 2017

David Franz Koza

Acknowledgments

I am a lucky person, for many reasons, and indebted to many people that made the last years an invaluable experience that I will not forget.

First of all, I want to thank my supervisors Stefan Røpke and David Pisinger for giving me the opportunity to pursue a PhD at the Operations Research group. Stefan Røpke, thanks for your trust, guidance and innumerable fruitful discussions during the past years. I learned a lot from you and feel prepared for what comes next. David Pisinger, I highly value your optimism, your attitude towards (real) problem solving and the fact that I could always count on your support if needed. It has been a privilege to work with both of you!

Professor Guy Desaulniers, thank you for hosting me at the Groupe d'études et de recherche en analyse des décisions (GERAD) in Montréal, for your engagement in our project and for your invaluable feedback.

I am thankful to all my former and current PhD fellows and my colleagues at DTU. You all have taught me a lot and you all have been an important reason for why I will keep this time in good memories.

Thanks to Mikkel Sigurd and the whole Maersk Line network planning team for the opportunity to gain practical insights into liner shipping and for sharing your experience with me.

Danken möchte ich auch meinem ehemaligen Lehrer, Herrn Becker. Ich habe lange nicht von mir hören lassen, aber nie vergessen, dass Sie bedeutenden Anteil daran haben, dass ich bis hierher gekommen bin.

Mama und Papa, ihr habt mir bei all meinen Entscheidungen vertraut und mich bei all meinen Vorhaben und Abenteuern unterstützt. Dank Euch hatte ich alle Möglichkeiten, von denen ihr viele selbst nicht hattet. Euch habe ich unendlich viel, wenn nicht alles, zu verdanken.

The greatest achievement during my PhD, after all, has been my marriage to my wife, Natalia. I thank you for your patience and for brightening every single day. This thesis would not have been possible without your unconditional support and encouragement. You are the most important of all reasons for why I am the lucky person that I am.

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Chapter 1

Introduction

Operations research is the discipline of ”applying advanced analytical methods to help make better decisions” (Informs, 2017). It is a scientific approach that combines different fields of research, including mathematics, statistics, computer science and engineering. Operations Research aims at analyzing and modeling complex systems and solving optimization problems using advanced mathematical methods. The ultimate goal is to help human decision makers make better decisions within complex systems and situations.

Container liner shipping is the primary mode of transportation of general cargo around the globe. Today’s liner shipping networks are considered the backbone of global trade, transporting more than 70% of the world trade in terms of value (WTO, 2008). The liner shipping industry – traditionally a rather conservative business – has seen stable and significant economic growth over many years with relatively low competitive pressure. The picture has changed in the first decade of the twenty-first century, though, and the industry is now struggling with lower growth rates, stricter environmental regulations and historically low cargo rates (Egloff et al., 2016). Finally, but later than other industries, liner shipping companies have started the process of *digitalization* – a term that describes the collection and use of (big) data and technology in order to improve business operations.

In this PhD thesis we develop and apply operations research tools to solve complex optimization problems that occur within the liner shipping industry.

Although breadth and depth of research addressing liner shipping has not reached the level of other industries like aviation or public transportation, operations research in liner shipping is, by no means, in its infancy anymore. During the past decades, various research groups around the globe have addressed a large number of different planning problems that occur on different planning levels in liner shipping. Research in the area has benefited greatly from collaborations with industry, as it made otherwise well-guarded data available to the research community.

The liner shipping industry, on the other hand, has reached a phase of consolidation after many years of steady above-average growth. As the liner shipping cake stops growing, having a larger slice of it has become more difficult. Besides the pressure from increased competition, the industry also faces exogenous challenges. Success stories from many industries have shown that analytics and operations research can help actors within liner shipping not only to mitigate risks but also to turn them into opportunities. And, from a practitioner's point of view, to be one step ahead of the competitors.

Although a solid foundation of research results has been laid for many operational, tactical and strategic planning problems, many gaps between theory and practice remain and new planning problems arise because of economic or legislative changes. In this thesis we present results that narrow some of the existing gaps, but also address recent and less explored challenges.

1.1 Thesis outline

This thesis consists of 5 chapters: *Introduction* (Chapter 1), *Integrated Liner Shipping Network Design and Scheduling* (Chapter 2), *Integrated Liner Shipping Network Design and Scheduling: Addendum* (Chapter 3), *Liner Shipping Service Scheduling and Cargo Allocation* (Chapter 4) and *The Liquefied Natural Gas Infrastructure and Tanker Fleet Sizing Problem* (Chapter 5).

Chapter 1 contains a brief introduction to the domain and operating principles of liner shipping, including the discussion of recent challenges and opportunities. The chapter includes a brief overview of liner shipping planning problems with references to selected papers and further outlines the contributions of this thesis. Chapters 2 to 5 constitute the core of the thesis and address different planning problems within or motivated by the liner shipping industry. Each of Chapters 2, 4 and 5 is a self-contained research article, whereas Chapter 3 presents and discusses extensions to the integrated liner shipping network design and scheduling problem.

1.2 Container liner shipping

Liner shipping networks constitute a key element in today's global supply chain. Indeed, around 70% of the world seaborne trade in value is transported by liner shipping companies (WTO, 2008). In Section 1.2.1 we first distinguish liner shipping from other modes of maritime transportation. The establishment and success of liner shipping have gone hand-in-hand with the process of containerization, which is described in Section 1.2.2. In Section 1.2.3 we introduce the domain and operating principles of liner shipping and present selected planning problems in Section 1.2.5.

1.2.1 Liner vs. tramp and industrial shipping

In maritime transportation, we usually distinguish between three modes of shipping (Lawrence, 1972), which are not necessarily mutually exclusive: Industrial shipping, tramp shipping and liner shipping.

In industrial shipping, the cargo owner and vessel operator are usually the same and the primary goal is to minimize transportation cost through vertical integration. The vessel fleet is commonly tailored towards transporting large volumes of a particular, often specialized type of cargo like, for example, liquefied gases, chemicals or refrigerated cargo. In tramp and liner shipping, vessel operators try to maximize profits by transporting goods of customers that pay for the transportation service. The main difference between tramp and liner shipping is their mode of operation. In liner shipping vessels sail on fixed routes following a published schedule and customers can book capacity on the vessels in exchange for paying a corresponding freight rate. In tramp shipping vessels serve origin-destination demands as contractually agreed. The two modes are often described as the equivalents to bus and taxi services in public transportation. For a broader introduction to maritime transportation and decision problems that arise within the three different modes of shipping, we refer the reader to Ronen (1983), Christiansen et al. (2004) and Christiansen et al. (2007).

In the remainder of this section, we will focus on container liner shipping. In Section 1.2.3, we provide a more detailed introduction into the domain of liner shipping and investigate attributes of and challenges faced by global liner shipping networks, as these are the main subject of application of this thesis.

1.2.2 Containerization

Containerization denotes the introduction and global adaption of standardized containers for transporting cargo. It allows to seamlessly move goods among ships, trains and trailers without the need of opening containers on their way between origin and destination. Empirical research suggests that containerization is not just a by-product of globalization, but actually one of its main drivers (Bernhofen et al., 2016).

The vast majority of containers have a length of either 20 foot (6m) or 40 foot (12m) and are commonly referred to as twenty-foot equivalent units (TEU) and forty-foot equivalent units (FEU). The height of most containers is either 8 feet 6 inches (2.6m) or 9 feet 6 inches (2.9m), with the latter known as *high cube* containers. In 2012, 85% of all containers were standard TEU, FEU or FEU high cube dry freight containers. The remaining 15% consists of refrigerated containers (*reefers*) and other specialized units like flat racks and tanks (Container Services International, 2012).

Before the age of containers, the handling of break-bulk cargo, e.g. in ports, required large numbers of workers, space and particularly time. The inherent ability of containers to be stacked has led to a significant reduction of space requirements, both on-shore and on vessels. The standardized shape of containers has further resulted in a high level of automation of cargo handling operations and heavily simplified the shift of cargo between modes of transportation. Whereas break-bulk ships spent around 50-70% of their time in ports, it is only 15-30% for container vessels (Christiansen et al., 2013) despite transportation volumes being much higher. Shorter cargo handling times reduced cargo transit times by 50% for a journey from Europe to Australia, for example, and with it inventory costs (Bernhofen et al., 2016). Not only did the total transportation time improve, but also its predictability. Widely used concepts like just-in-time manufacturing would not be possible without containerization (Levinson, 2006). As containers do not have to be opened for cargo handling, the risk of theft decreased significantly as well, and so did insurance costs (Bernhofen et al., 2016). To summarize, containerization has made transportation faster, more economical and more secure. Economist Levinson (2006) devotes a whole book to ”the box” and provides a well-investigated exploration of the history of containerization.

From the shipping market's point of view, containerization has led to a high level of *commoditization* of transportation services. Products of different attributes like shape or value become indistinguishable if transported in containers. Differentiation of transportation services has become much more difficult and as a result, freight rates are the by far most important decision criterion for customers of liner shipping companies. In Section 1.2.4 we will elaborate further on the challenge of product differentiation faced by liner shipping companies.

Liner shipping is responsible for the transportation of the vast majority of containers. Thus, the success story of containerization is the success story of liner shipping. The next section provides an introduction to the domain and operating principles of liner shipping networks.

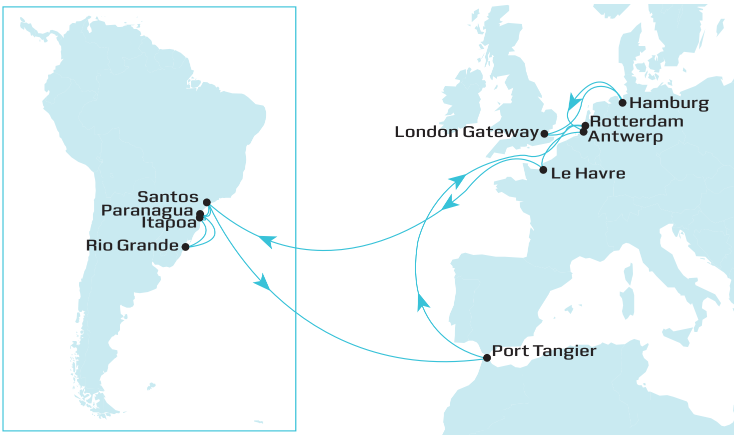


Figure 1.1: A Maersk Line service connecting Europe and Brazil (Maersk Line, 2017). The planned arrival and departure times and sailing times are reported in Table 1.1.

1.2.3 Industry characteristics

Liner shipping networks consist of multiple *services*. A liner shipping service is a *round trip* that is operated at a fixed frequency following a published schedule. A round trip consists of a sequence of *port calls*, i.e. stops at ports where the vessel can load and unload containers. Commonly, a route has no defined start or end, because a vessel can load and unload cargo at each port. Figure 1.1 shows a Maersk Line service that operates between Europe and Brazil. Some services are split into *headhaul* and *backhaul* voyages, with headhaul denoting the direction of strong transport demand and high capacity utilization. An example is the Asia-Europe trade (headhaul), where demand is traditionally much higher than on the backhaul voyage from Europe to Asia.

The majority of services are operated at a weekly frequency. Each port on the service is called once per week in that case, generally at the same time of the week. The port call times for each port are published and available to customers. Table 1.1 shows the public schedule of the service shown in Figure 1.1. Customers, also called *shippers*, thus know a service's weekly departure time and the corresponding *cut-off* time, i.e. the latest time cargo can be delivered to the terminal to be picked up by the next scheduled vessel. The regularity of liner services facilitates planning for shippers. The total round trip time of a service can be up to multiple weeks. To ensure a weekly frequency, the number of vessels that operate a service equals the service duration in weeks. The Maersk Line service shown in Figure 1.1, for example, has a total duration of seven weeks, and around 12 weeks are needed for a round trip from Asia to Europe and back.

Generally, all vessels deployed on a service are of the same or similar capacity to guarantee that the loading capacity does not fluctuate too much between weekly arrivals. The vessel *fleet*,

Port	Country	Arrives	Departs	Transit
Rotterdam	Netherlands	Wed	Fri	–
London Gateway	United Kingdom	Fri	Sat	2
Hamburg	Germany	Sun	Mon	4
Antwerp	Belgium	Tue	Wed	7
Le Havre	France	Thu	Fri	8
Santos	Brazil	Wed	Fri	22
Paranagua	Brazil	Sat	Sun	24
Rio Grande	Brazil	Sun	Mon	32
Itapoa	Brazil	Tue	Wed	33
Paranagua	Brazil	Wed	Thu	34
Santos	Brazil	Sat	Sun	36
Port Tangier	Marroco	Sat	Sun	45
Rotterdam	Netherlands	Wed	Fri	49

Table 1.1: Schedule of the Maersk Line service shown in Figure 1.1, with a total duration of 49 days (Maersk Line, 2017).

i.e. the total pool of vessels operated by a large company, can exceed several hundred ships. The ten largest liner shipping companies operate between 80 and 660 vessels each. The average vessel capacity of all ships among the ten largest operators is 6.000 twenty-foot equivalent units (TEU), with the largest ships exceeding 20.000 TEU of nominal capacity. Around 37% of the vessels are owned by the companies and around 63% are chartered (Alphaliner, 2017).

The cost of operating a service consists of different cost components. The weight of each cost component can vary significantly depending on the vessel type, bunker prices, charter rates and also the region of operation. The capital or charter costs, the bunker fuel costs, the cargo handling costs and the fees for port calls and canal passages constitute the largest cost components. Figure 1.2 shows different cost components for different vessel sizes on a transpacific service that connects China and Japan with the US west coast. It illustrates the cost savings that can be achieved through economies of scale. The unit fuel costs, the unit capital cost and also the unit operating cost are significantly lower for larger vessels. Note that cargo handling costs are independent of the vessel size, as terminals charge vessel operators per container loaded or unloaded. The unit port costs, representing the fixed cost associated with calling a port, again decreases with larger vessel sizes. Figure 1.2 also shows that the marginal cost saving per extra unit of capacity decreases for larger vessels, i.e. the curve gets flatter.

Whereas larger vessels have a clear advantage on the unit cost side, the increased vessel size also has its downsides (OECD, 2015b). Some ports simply do not have the necessary depth or infrastructure to accommodate or efficiently handle very large vessels. Due to the larger number of container loading and unloading operations per port call, the turnaround time of large vessels is significantly longer and the number of port calls on a route likely to get reduced. As the lower unit cost can only be fully exploited at a sufficient capacity utilization, cargo will necessarily have to be consolidated along the routes that are operated by large container vessels.

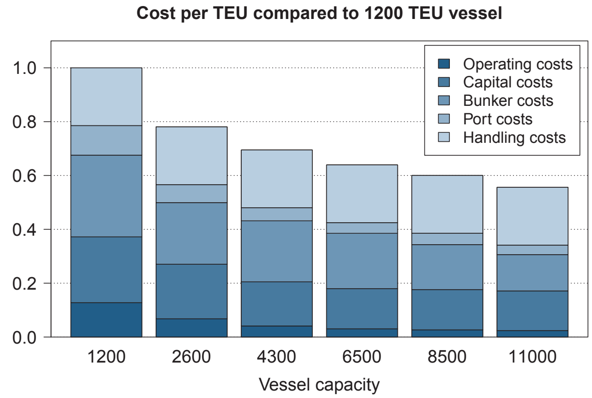


Figure 1.2: Comparison of different cost components and different vessel sizes on a transpacific service that operates between China/Japan and the US west coast. The total unit cost of the smallest vessel type is normalized to one and all other unit costs are relative to that of the smallest vessel type (own composition based on data from Stopford, 2009)

Global and strategic liner shipping *alliances* are, to some extent, a consequence of increasing vessel sizes and the need to fill them. Alliances represent co-operative agreements between different liner shipping operators with the goal of achieving operational synergies. Different forms of cooperation exist. These are, for example, the common deployment of shared vessels on a shared service (*vessel sharing*), making capacity accessible to cooperation partners (*slot sharing*) or sharing terminal capacities, among others. The alliances allow operators to offer a larger number and variation of routes and schedules to their customers and to achieve higher utilization rates through collaboration. On the other hand, each participating carrier usually remains responsible for its own pricing, marketing and assets (Panayides and Wiedmer, 2011). In 2017 three large alliances (of formerly four) remain (see Table 1.2).

The networks of large liner shipping companies, within an alliance or alone, consist of a large number of services that are operated by several hundred vessels. These networks cover a large number of ports around the globe and allow shippers to send containers between almost any possible pair of ports. The majority of origin-destination pairs, however, is not connected directly. Instead, containers can be moved between services at ports that are visited by both the

Alliance	**Participating operators**	**Ranks of operators**	**Market share**
2M	Maersk, MSC	1st, 2nd	31.2%
Ocean	CMA, COSCO, Evergreen, OOCL	3rd, 4th, 6th, 7th	28.2%
THE	Hapag-Lloyd, Yang Ming, MOL, NYK, K Line	5th, 8th, 9th, 11th, 15th	16.6%

Table 1.2: Liner shipping alliances, their members, member ranks by capacity and aggregate market share (of total TEU capacity, based on data of Alphaliner, 2017).

Figure 1.3: Visualization of Maersk Line's global liner shipping network (`https://www.maersk.com/en/explore/fleet`)

unloading and the loading service. The movement of a container from one service to another is called a *transshipment*. In fact, transshipments are what turns a set of individual liner services into an interdependent network and thus represent a key aspect of global networks. The exchange of cargo between services does not have to happen simultaneously, but containers can be stored at a port until being picked up by the loading service. Figure 1.3 visualizes the network of Maersk Line and gives a good impression of the complexity of such a network. In global liner shipping networks, transshipments occur very frequently. In the network of Maersk Line, for example, more than half of all containers are transshipped at least once on their journey from the origin port to the destination port (Karsten, 2016).

From a shipper's point of view, the transportation service offered by liner shipping companies consists of different components. Besides the most evident one, the charged freight rate, various other characteristics may matter to the customers (Stopford, 2009; Brouer et al., 2014). These characteristics can be divided into quality of service and reliability of service. The quality of service is characterized by the offered frequency of sailing, the general availability of transport capacity, the origin-destination transit time and the number of transshipments, but also by less obvious components like cargo tracking abilities for shippers. The service reliability relies on the liner shipping company's schedule adherence, but also on its responsiveness to customer requests and problem-solving performance in case of unexpected disruptions. Last but not least, the network coverage of an operator can be decisive about whether a customer can actually use a particular company's network to deliver cargo or not.

1.2.4 Challenges and opportunities

After years of steady growth, the liner shipping industry currently is and will be undergoing changes driven by endogenous and exogenous challenges. In this section, we briefly discuss some of the problems faced by liner shipping companies, such as the slowdown in demand, growing overcapacity but also legislative challenges like stricter environmental regulations.

Opportunities arise from the fact that large liner shipping networks are huge, complex systems that are difficult to optimize holistically. Combined with the fact that the industry has traditionally been rather conservative in terms of adaption of new technologies, there exists an enormous potential for improving cost, efficiency and quality. The collection and use of data, nowadays termed *digitization* and *digitalization*, represents such an opportunity that the industry has started to take advantage of and will be briefly discussed as well.

Market slowdown The liner shipping business has traditionally experienced growth rates far above the growth of the world gross domestic product (GDP); between 1983 and 2006, the volume of containerized cargo grew, on average, by 10% per year whereas the world GDP increased by 4.8% per year in the same period (Stopford, 2009). The growth was driven by two factors: First, cargo that was previously transported as bulk cargo was *containerized*, i.e. existing non-containerized demand turned into demand for containerized transportation. Second, the process of globalization came along with a strong increase in world trade volumes.

With almost all goods that can be transported in containers being containerized by now, the future demand for containerized seaborne transportation will mainly rely on the trade volume growth at sea (Christiansen et al., 2013). The future prospects for the latter have also become less optimistic, as offshoring of production to countries with lower labor costs is slowing down as well (Sanders et al., 2015). And indeed, in 2015, and for the first time in history, the growth in container demand was lower than the global GDP growth. Rather than being a temporary phenomenon, the flattening of container transport demand is expected to persist (Egloff et al., 2016).

Ultra large container vessels and overcapacity Sanders et al. (2015) identify only two carrier types – global scale leaders and niche specialists – as profitable. The former are large carriers following rigid rationalization and cost reduction programs to keep unit costs low, and the latter are smaller but specialized operators that focus on niche markets with higher profit margins.

Indeed, in order to achieve economies of scale, a large number of carriers has invested in significant numbers of ultra large container vessels (ULCV), the largest of which exceed capacities of 20.000 TEU. Today, the capacity of the largest vessels is four times the size of those in the Panamax era. The advantages of ULCVs are hard to deny at first sight. Larger vessels have a significantly lower unit capital cost, unit operational cost and unit fuel cost. The strive for economies of scale has resulted in a doubling of average container vessel capacity between 2000 and 2014 (Tran and Haasis, 2015a).

However, the lower cost per unit transported only pays off if the utilization does not deteriorate. To keep load factors stable, either demand has to increase substantially or liner shipping companies have to consolidate more cargo along the trades on which the ULCVs are deployed. For shippers, increased consolidation of cargo often implies less direct shipping routes and hence a lower quality of service.

The increase of total TEU capacity has outperformed the growth in transport demand in the past decade. To achieve higher utilization levels, freight rates have dropped significantly. The orders for ULCVs, however, have remained high, as companies intend to reduce unit transportation costs to remain competitive and to cope with lower freight rates. This development has resulted in a vicious cycle hard to break for a single company. As a consequence, many actors in the liner shipping market have been making losses (Sanders et al., 2015).

Market consolidation The combination of a market slowdown and significant overcapacity has put enormous cost pressure on container lines. As a consequence, the liner shipping market has undergone heavy consolidation over the past years. Alone in 2016 and 2017 multiple major mergers and acquisitions took place among the largest liner shipping companies. In 2016 CMA CGM acquired Neptune Orient Lines and the two Chinese carriers COSCO and CSCL merged. Hapag-Lloyd merged with United Arab Shipping Company in 2017 and market leader Maersk Line is finalizing the acquisition of Hamburg Süd, which was among the ten largest container lines before the purchase. The three Japanese liner shipping companies NYK, MOL and K line, all among the largest 15 container liners, also announced plans for a merger in 2016. And most notably, the South Korean Hanjin Shipping company, formerly among the ten largest liner shipping companies, was declared bankrupt in 2017. It is considered the largest and most significant bankruptcy in the industry.

The ongoing market consolidation is reflected in the concentration of TEU capacity among a small number of large companies. In 2001 the 16 publicly traded liner shipping companies represented 34% of the global TEU capacity and this value had grown to 57% in 2011 (Tran and Haasis, 2015a). Today, half of the total TEU capacity is deployed by the top five container lines and the top ten accumulate more than 75% of total TEU capacity (Figure 1.4). And these numbers do not reflect all of the latest mergers and acquisitions yet.

As a result, large liner shipping networks have to grow together and result in even more complex systems. The integration of formerly independent services or networks into a large operating network may result in overlaps and internal cannibalization if not revised and restructured properly. In Chapter 2 we present a model and algorithm that allow to (re-)design networks partially and under full consideration of existing services and their schedules. The model and method presented in Chapter 4 does not alter the network, but reschedules existing services to

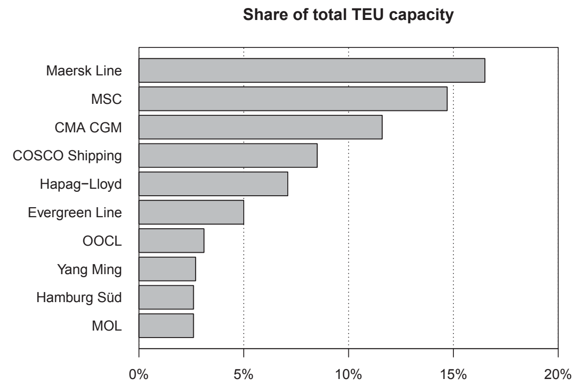


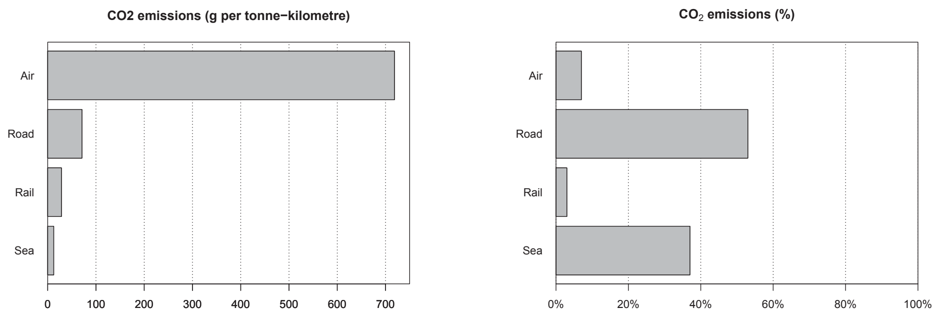
Figure 1.4: Market share of the ten largest liner shipping companies in terms of TEU capacity (own Figure based on data from Alphaliner, 2017). The 2017 acquisition of Hamburg Süd through Maersk Line is not reflected in the data yet.

achieve an optimal synchronization that minimizes operational costs without deteriorating the service level.

Product differentiation Containerization of cargo has led to a high level of commoditization of transportation services. From a liner shipping company's point of view, it is not particular goods of particular value that have to be transported, but simply containers. As a result, the cost of transportation has become the most important decision criterion from a shipper's point of view. Transportation services can still, however, differ between, but also within liner shipping companies. Attributes besides the pure freight rate are transit times, the number of transshipments between origin and destination port and the reliability with respect to promised transportation times.

In 2011 Maersk Line started an attempt of product differentiation by introducing a premium product called "Daily Maersk" (Maersk Line, 2011). It allowed shippers to deliver export cargo intended for shipment to Northern Europe on a daily basis at selected ports in Asia. The daily cut-offs allowed customers to immediately ship cargo after production without the need for storage. Maersk Line promised agreed pick-up times and fixed transportation times independent of the weekday of delivery at the pick-up port. The company would further compensate shippers in case containers arrive late their destination port. In return for guaranteed delivery times and daily cut-offs, shippers had to pay higher freight rates for the premium product. In 2015, Maersk Line basically stopped offering the service, stating that it was not a commercial success because customers were not willing to pay for better quality services (Porter, 2015).

The previous example shows how dominant the transportation cost is as a decision criterion for shippers. Nevertheless, it would be rash to conclude that the service level is irrelevant to



(a) CO_2 emissions per tonne-km and freight transport mode (own estimates based on McKinnon, 2008).

(b) Share of total CO_2 emissions per freight transport mode (OECD, 2015a).

Figure 1.5: CO_2 emissions per freight transport mode.

customers. In fact, liner shipping companies do take measures to improve transit times in order to attract shippers, but under the challenging premise of not deteriorating cost performance. The 2M alliance of Maersk and MSC, for example, has reduced the number of port calls along the Asia-Europe trade in order to offer better transit times without the need for burning more fuel by speeding up their vessels (King, 2016). It comes, however, at the expense of less direct connections and again raises the need for consolidation of cargo. The example reflects the difficulties and complex trade-offs that liner shipping network operators face under recent market conditions.

Both the tactical model and approach to (re-)design liner shipping networks in Chapter 2 as well as the solution approach for the service scheduling and cargo routing problem in Chapter 4 aim at the reduction of operating costs while accounting for service level requirements.

Environmental regulations Shipping as a mode of freight transportation is commonly considered as clean when compared to other modes of transportation. Indeed, the average CO_2 emissions per tonne-kilometer are significantly lower for sea transport (10-15 g/tonne-km) than for rail (19-40 g/tonne-km), road (51-91 g/tonne-km) or air (570-867 g/tonne-km) transport (Figure 1.5).

However, with 85% the large majority of global trade in tonne-km is carried by sea and sea transportation accounts for more than one-third of all trade-related CO_2 emissions. In absolute terms, only road transportation is a larger emitter of CO_2 (Figure 1.5b). The emissions caused by shipping are expected to increase significantly by 240% between 2010 and 2050 (OECD, 2015a).

Due to the international nature of shipping, the enforcement of regulations requires international institutions. In 1959 the predecessor of today's International Maritime Organization (IMO) was founded with the goal of regulating shipping. Today, with the exception of a few countries, of which the majority are landlocked, almost all UN member countries are also members of the IMO. The IMO is a specialized agency of the United Nations that aims at creating a regulatory framework for the shipping industry, covering a wide range of aspects like safety, environmental issues, security or legal affairs (International Maritime Organization, 2017).

In 2008 the IMO approved a set of regulations to prevent or significantly reduce airborne emissions caused by ships (IMA, 2008). Since 2015 the regulations were first applied in *emission control areas* in the North Sea, the Baltic Sea and North American coastal areas and will become binding globally in 2020. The regulation defines, among others, a stricter limit on the permitted sulphur oxide emissions from ship exhaust which many ships would violate if operated as of today. The IMO regulations have drawn the industry's attention to alternative, but cleaner fuels like liquefied natural gas. A transition from heavy fuel oil fuels to alternative fuels is, however, not trivial as it requires newbuild vessels and the buildup of new on-shore infrastructure. In Chapter 5 we present a case study with a large liner shipping company that considers the use of liquefied natural gas (LNG) as an alternative fuel. The article addresses a large-scale LNG infrastructure and tanker fleet sizing problem with the goal of minimizing combined long-term investment and operational costs.

Digitization and digitalization The terms *digitization* and *digitalization* have turned into popular buzzwords and some expect the underlying ideas to revolutionize the way business is done. Either way, digitization and digitalization have gained attention in the liner shipping industry as well. Some of the headlines in the year 2017 read: "Maersk goes big in digital transformation with Microsoft" (Microsoft, 2017), "MSC Embraces the Digital Age" (Port Technology, 2017) or "CMA CGM invests in digital transformation" (King, 2017).

Whereas the two words sound quite similar, the distinction is important: Digitization describes the process of converting analog data into digital data and the automatic collection of digital data. Digitalization, on the other hand, aims at turning digitized information into knowledge and at using data to improve a business.

The container shipping business has traditionally been a rather conservative business when it comes to the adaption of new technologies. The article with the above-cited headline "MSC Embraces the Digital Age" is from May 2017 and reports that Mediterranean Shipping Company (MSC), the world's second largest liner shipping company, has started a collaboration with a social media consultancy in order to start communicating with customers online. And indeed, most of the communication with customers is still done through phone or fax and most of the documentation of a shipment and related processes is still done on paper. However, other liner shipping companies have taken measures that aim at *digitization*, i.e. the transformation of analog data into digital data. It is not only expected to heavily reduce paperwork and related costs, but also to make data available both to the liner shipping company as well as to the shippers. In its annual report 2016, A.P. Møller-Mærsk (2017) even considers digitization as the industry's second major challenge besides oversupply and resulting low freight rates. The other two articles cited at the beginning of this section report efforts made by large companies to push the digitization process. An important result of this process is the growing availability of data for both shipping companies as well as shippers and, last but not least, operations research analysts.

And this is where *digitalization* comes into play. As Meng et al. (2014) note, companies like Maersk Line and Orient Overseas Container Line (OOCL) have started to collaborate with the research community. The work presented in this thesis, particularly in Chapters 2 to 4, is part of the digitalization process that aims to turn data into better decisions using analytics and optimization tools.

1.2.5 Planning problems in liner shipping

In the past decade the number of publications addressing maritime optimization problems has grown substantially, both in breadth as well as in depth. This is very well reflected in the number and scope of literature reviews published during the past years: Christiansen et al. (2007) provide a broad overview of models and methods in maritime transportation. Christiansen et al. (2013) review literature in the field of ship routing and scheduling and Psaraftis and Kontovas (2013) review and classify contributions in ship speed optimization. Meng et al. (2014) provide an overview of ship routing and scheduling exclusively for applications in liner shipping. The review by Tran and Haasis (2015b) has an even narrower scope, focusing on network design within liner shipping. These review articles represent an excellent starting point for further literature research.

In the remainder of this section we briefly introduce some of the many planning problems that arise within liner shipping. These include cargo allocation problems (Section 1.2.5.1), ship routing problems (Section 1.2.5.2), vessel speed optimization problems (Section 1.2.5.3), scheduling problems (Section 1.2.5.4) and liner shipping network design problems (Section 1.2.5.5). All of these problems are addressed either directly or indirectly in subsequent chapters of this thesis.

In each subsection we provide selected references, which, however, are by no means exhaustive. Each of the articles in this thesis contains a separate review of literature relevant to the problem studied.

1.2.5.1 Cargo allocation

The cargo allocation or cargo routing problem (used synonymously in the following) is an operational planning problem that liner shipping companies face on a daily basis. Particularly in global networks many viable origin-destination paths may exist. These can differ in transit time and the number of transshipments. From the carrier's perspective the problem allows to steer vessel utilization. Low-value cargo may be allocated to services of low utilization to avoid blocking capacity along routes of high demand.

A variation of cargo allocation or routing problems are empty container repositioning problems. They arise from the imbalances of trade along many container transportation routes. On major container transportation markets like Asia-Europe or Pacific, headhaul trade was more than twice as large as backhaul trade in 2015 (Waters, 2016). The repositioning of empty containers does not generate any revenue, but implies cost for transportation.

Brouer et al. (2011) present a cargo routing problem that explicitly considers the repositioning of empty containers. Wang et al. (2013) address an origin-destination cargo routing problem that considers transit time restrictions as well as cabotage rules. Karsten et al. (2015) solve the cargo routing problem under transit time restrictions for whole liner shipping networks using a column generation algorithm. In a large number of papers cargo routing problems occur in combination with other problems and are often solved to evaluate schedules, routes or networks. In Chapters 2 and 4 we describe and use two improved version of the algorithm of Karsten et al. (2015) to solve cargo allocation problems that arise as subproblems in a network design and in a service scheduling problem.

1.2.5.2 Ship routing

Ship routing problems are among the most common optimization problems addressed in the maritime transportation literature (see e.g. the review by Christiansen et al., 2013) and arise on different planning levels as well as contexts. We give a few examples for routing problems in liner shipping.

Fagerholt (2004) considers a container ship routing problem motivated by a real maritime transportation problem faced by Norwegian shippers. The goal is to design weekly reoccurring routes for container vessels that transport cargo from production ports to a hub. Plum et al. (2014) address the problem of designing a single liner service under minimization of operational cost and maximization of revenues from transporting demands between ports. Transit time restrictions on the demands and vessel capacity constraints have to be respected.

The paper by Tierney et al. (2015) addresses a fleet repositioning problem in liner shipping that arises when vessels are (re-)assigned to new or different services. The objective is to perform the repositioning of vessels such that additional operational costs but also disruptions to daily operations are minimized. This includes the reallocation of cargo, which has to reach its destination despite the repositioning of vessels.

Whereas routing problems are generally considered tactical problems (Meng et al., 2014), they may also occur in operational settings as, for example, in the event of disruptions. Brouer et al. (2013) and Li et al. (2015), for example, describe disruption recovery problems. The re-routing of vessels is one of the feasible actions to get the delayed vessels back to schedule.

1.2.5.3 Speed optimization

An extensive survey and taxonomy of maritime transportation models that include speed optimization are presented in the work by Psaraftis and Kontovas (2013). Psaraftis and Kontovas (2014) discuss the most important aspects that may influence speed or routing decisions from a ship owner's or charterer's point of view. Speed optimization models that consider payload dependent fuel consumption, cargo inventory costs and freight rates are presented for a single ship setting.

Some papers on speed optimization consider particular economic or legislative contexts. The paper by Fagerholt et al. (2015), for example, investigates how stricter environmental regulations and the resulting increase of fuel costs within emission control areas affects optimal routing and speed decisions. Notteboom and Vernimmen (2009) assess the impact of increased bunker costs on the operation of liner services. The paper discusses countermeasures undertaken by liner shipping companies to cope with increased bunker costs, including the adaption of speed.

Guericke and Tierney (2015) and Karsten et al. (2016) consider combined speed optimization and cargo routing problems within liner shipping networks. The presented problems aim at fuel cost minimization through speed optimization while simultaneously determining the routing of cargo through the network. Maximum transit times are defined for cargo, resulting in a trade-off between fuel cost savings from speed reduction versus lost revenue due to increased cargo transit times.

1.2.5.4 Ship scheduling

Scheduling problems aim at finding feasible schedules for vessels. Various papers in the maritime transportation literature address ship routing problems under consideration of time windows, making the schedule a crucial part of a solution Wang et al. (2014), Fagerholt (2001), and Halvorsen-Weare and Fagerholt (2013). The time window constraints may reflect different practical restrictions, as for example restricted port berth availabilities or contracts that specify delivery time windows.

In liner shipping networks, service schedules are particularly relevant for transshipment operations. The port call times of services determine the transshipment times between two services at a port. Moreover, scheduling problems in liner shipping are often related to speed optimization problems, particularly if fuel cost minimization is part of the objective. In fact, scheduling can be seen as a generalization of speed optimization, because any vessel schedule implicitly defines a (minimum) speed profile for the same vessel. Reinhardt et al. (2016) and Wang and Meng (2012) address similar berth scheduling problems within liner shipping networks with the purpose of vessel speed optimization. The problems aim at fuel cost reduction through speed optimization while complying with maximum container transport times along fixed routes in liner shipping networks. Different to the speed optimization models of Guericke and Tierney

(2015) and Karsten et al. (2016), their models define schedules that allow to calculate exact transshipment times of containers between services.

In Chapter 4 we consider a combined service scheduling and cargo routing problem.

1.2.5.5 Liner shipping network design

Liner shipping network design problems are at the core of liner shipping planning problems and address the question of how to optimally construct a network of services. Network design problems are among the most difficult maritime optimization problems, as they represent a combination of multiple other difficult planning problems. These include, for example, fleet deployment, ship routing, cargo allocation and schedule design or speed optimization problems. Most commonly the objective is to minimize investment and operational costs for operating liner services and simultaneously maximize revenues generated from transporting containers between ports.

There are several important contributions to liner shipping network design problems. The work by Agarwal and Ergun (2008) is considered to have established liner shipping network design problems as its own class of maritime optimization problems. Brouer et al. (2014) developed a publicly available library of liner shipping network design problem instances (named LINER-LIB) and thus made the problem available to a broader audience. Various contributions have emerged on the basis of LINER-LIB instances, most notably a series of contributions that explicitly consider the service level of a network (Karsten et al., 2016, PhD thesis). Other contributions are motivated by particular case studies or address variations of the problem (for example Álvarez, 2009; Mulder and Dekker, 2014; Meng and Wang, 2011).

In Chapter 2 we discuss existing contributions in more detail and describe a rich liner shipping network design and scheduling problem together with a novel column generation based solution algorithm to solve it.

1.3 Conclusion

Operations Research in maritime transportation has evolved into a broad and active field of research over the last decades, and liner shipping has established itself as one major branch within this field. The number and variety of decision problems that arise within liner shipping is vast and keeps growing due to the economic and legislative changes that the industry is undergoing. Although later than other industries, actors within the industry have sought collaboration with academia, and these collaborations have enabled researchers to lay the foundation that this thesis builds upon. We hope that this thesis will form the foundation for future research within the field in the same way.

This thesis presents models and methods to design and support large liner shipping networks. The presented work contributes to the field of Operations Research in general and to applications of Operations Research in liner shipping in particular. The contributions of each chapter are detailed in Section 1.3.1. In Section 1.3.2 we list conferences and workshops at which research results contained in this thesis have been disseminated and shared with the Operations Research and (maritime) transportation community.

The contributions of this thesis bridge some of the existing research gaps, but other gaps remain. In Section 1.3.3 we discuss possible avenues for future research.

1.3.1 Contributions

The main scientific contributions of this thesis are presented in three self-contained articles in Chapters 2, 4 and 5. In addition, Chapter 3 describes and discusses various extensions to the article in Chapter 2. In the following we outline the contribution of each of the chapters in this thesis.

In **Chapter 2** we formulate and address a problem that integrates the liner shipping network design problem with the liner shipping scheduling problem, hereafter called the **liner shipping network design and scheduling problem** (LSNDSP). The liner shipping network design problem alone is a classical strategic planning problem in liner shipping. It is a rich but difficult optimization problem that combines various other planning problems, many of which are difficult problems themselves.

The integration of network design and scheduling decisions moves the classical network design problem closer to industrial reality. While it is very unlikely that a container carrier re-designs its network from scratch, partial modifications do occur very frequently. The most common reasons are seasonal or fundamental changes in market conditions that require liner operators to redesign or reschedule single or multiple services. The acquisition of new vessels and/or the de-commissioning of older vessels lead to changes on the supply side and usually require adaptions of the network, if the cargo carrier wants to maintain high utilization rates. Larger modifications do occur as a consequence of the ongoing consolidation in the container shipping sector, resulting in mergers or acquisitions. The announcement of alliances or vessel sharing agreements may require the participating liner shipping companies to adjust routes and schedules in order to offer the most economic and fastest port-to-port transport connections. What the described scenarios have in common is that they cannot be treated independently, but they require the integration and synchronization with the remaining network to offer fast and economic transportation connections for shippers. The scheduling decisions are of crucial importance under the discussed scenarios, because they determine the transshipment times of containers between individual liner services.

The model and method presented in the article allows decision makers to design or improve liner shipping networks or parts of it, while fully accounting for network effects. Besides the integration with scheduling decisions, the presented work further combines various modeling features that previously have only been considered alone. These are, among others, the consideration of service level requirements and the ability to model any type of complex liner shipping service. The presented work narrows the gap between the mathematical modeling of liner networks and the economic and operational reality of liner shipping substantially. Additionally, the problem is shown to be $\mathcal{NP}$-hard in the strong sense by reduction from the asymmetric traveling salesman problem.

The work further contributes with a novel column generation based matheuristic to solve the liner shipping network design and scheduling problem. The method scales well to larger instances despite the significantly increased complexity of the problem. Indeed, it finds new best solutions to all four considered instances of the public LINER-LIB benchmark suite if compared under equal restrictions. The presented matheuristic combines and extends advanced linear programming techniques and we believe that the developed framework constitutes a promising solution framework for the wide class of large-scale service network design problems.

Chapter 3 presents various **extensions to the liner shipping network design and scheduling problem** and describes how these can be integrated into the model presented in the previous chapter. The extensions cover relevant issues as, for example, empty container repositioning, the explicit consideration of refrigerated containers and the incorporation of more complex revenue functions. Moreover, we briefly introduce a more general problem formulation for the liner shipping network design and scheduling problem that allows to incorporate an even richer set of practically relevant restrictions on the design of liner services.

The problem we address in **Chapter 4**, the **liner shipping service scheduling and cargo allocation problem** (LSSCAP), is closer to operations and aims at simultaneously optimizing service schedules and cargo routing decisions. The two tactical and operational planning problems are interdependent in practice; the schedules are commonly designed to offer fast transportation routes between ports and, conversely, the optimal routing of cargo depends on the schedules of the liner services. The article presents a novel branch-and-price solution algorithm with an integrated primal heuristic that quickly generates good feasible schedules and cargo routing plans. The consideration of this problem was partly motivated by the findings in Chapter 2, but mainly by the fact that the two problems have not been considered together despite their close relation.

The second important contribution is the explicit consideration of payload in the vessels' fuel consumption functions. Although neglected in the majority of studies that address speed optimization in liner shipping, the payload can have a significant influence on a vessel's fuel consumption. The article further derives parameters of payload dependent fuel consumption functions for all LINER-LIB vessel classes to encourage researchers to use more realistic fuel consumption functions.

The analysis of computational experiments allows to draw important conclusions. First, the results show that ignoring a vessel's payload when optimizing sailing speed can indeed result in suboptimal schedules and sailing speed profiles. Second, the findings underline the importance of modeling the service schedules and resulting transshipment times explicitly. The results show that not modeling schedules and instead approximating transshipment times between services is likely to result in significant under- or overestimation of cargo transit times, depending on the chosen approximation value.

The problem presented in **Chapter 5** is part of an industrial case study conducted with a large liner shipping company. It considers the use of an alternative fuel, liquefied natural gas (LNG), for container vessels along a major trade lane. The challenge to establish the necessary on-shore infrastructure and to purchase and operate tankers to distribute the LNG is termed the **liquefied natural gas infrastructure and tanker fleet sizing problem**. The case study arose out of the decision of the International Maritime Organization to impose stricter limits on airborne emissions of vessels. The new regulations become binding in 2020 and are expected to have significant impact on the operating principles in liner shipping.

The article addresses an unstudied problem and presents different model formulations to formalize it. The study further contributes with an extensive sensitivity analysis based on different case scenarios and analyzes the interaction between long-term investment and operational costs. The results permit to draw up a set of decision rules and recommendations valuable to the decision maker of the considered large scale LNG supply infrastructure problem.

1.3.2 Dissemination

In the following a list of conferences and workshops where we presented research results obtained during the PhD studies. Please note that not all the presented work has finally resulted in written articles.

- D.F. Koza, S. Røpke, A.B. Molas. *The liquefied natural gas infrastructure selection and tanker routing problem - A case study.* European Conference on Operational Research, Glasgow, 12-15 July 2015
- D.F. Koza, L.B. Reinhardt, S. Røpke. *The liner shipping network speed optimization problem.* 2016 Optimization Days, Montréal, 2-4 May 2016
- D.F. Koza, S. Røpke. *The liner shipping speed optimization problem and a heuristic solution method.* Ninth Triennial Symposium on Transportation Analysis (TRISTAN IX), 13-17 June 2016, Oranjestad, Aruba
- D.F. Koza, G. Desaulniers, B. Brouer. *The Liner Shipping Network Design Problem.* EURO Winter Institute "Methods and Models in Transportation", Bressanone, 14-23 February 2017

- D.F. Koza, G. Desaulniers, S. Røpke. *Integrated Liner Shipping Network Design and Scheduling.* 21st conference of the International Federation of Operational Research Societies (IFORS), Quebec City, 17-21 July 2017

1.3.3 Future research directions

The models and methods presented in this thesis bridge important gaps. Nevertheless, room for extensions always remains as well as opportunities to transfer knowledge to other areas of application. In this section we discuss selected potential future research avenues that relate to the presented work and that we believe are promising.

The liner shipping network design and scheduling problem addressed in Chapter 2 can be extended in many ways. In Chapter 3 we present some of these extensions and detail how to model and implement them. We further present a service-based formulation of the problem that allows to model a much richer set of restrictions for the generation of liner services. As briefly discussed, these restrictions may require or forbid particular sailings, require or forbid particular orders of port calls or impose a minimum or maximum duration for a service, to name a few. Basically, any restriction that can be expressed logically could be imposed. Whereas the resulting model constitutes a rich generalization of a wide class of service network design models, it comes at the expense of additional service pricing problems that need to be solved to generate new services. Special cases of these have been addressed in the literature (see e.g. Andersen et al., 2011). The proposed methods, however, rely on the specific structure of these special cases and are likely to not scale well for more general pricing problems. Furthermore, the structure of the (master) problem would change, requiring a reevaluation and recalibration of the method proposed in Chapter 2. However, the development of a solution algorithm for the service-based model formulation based on the method for the original model represents a worthy research direction, as it would provide a tool to solve a wide and rich class of service network design problems.

One important contribution of the article that addresses the LSSCAP (Chapter 4) is the explicit consideration of payload in the fuel consumption function. The same approximation technique can be used in the model for the LSNDSP (Chapter 2). However, as the underlying graph models differ, the use of a payload dependent fuel consumption function in the model and solution method for the LSNDSP is not straightforward but would make the problem more difficult to solve. Whether the increased difficulty actually affects computational runtimes needs to be investigated.

Delays and disruptions are no exceptions in liner shipping. Whereas they are commonly difficult to forecast, liner shipping operators can indirectly mitigate the negative effects by including buffer times in their network schedules as a proactive measure. The right amount and geographic assignment of schedule buffer times is not trivial and would depend on the expected number and severity of disruptions in the network. Forecasting and solution methods for optimal buffer time allocation constitute a practically relevant research area and represent a possible extension to the model and method for the LSSCAP (Chapter 4).

The positive results obtained for very large LSNDSP instances (Chapter 2) encourage to adapt the developed algorithm framework to other problem classes, as for example capacitated network design problems or, again, service network design problems. We believe that algorithm components like the dual variable penalization procedure may improve the performance of algorithms for variations of capacitated network design problems in the same way as they do for the LSNDSP.

The model for the strategic LNG infrastructure and tanker fleet sizing problem addressed in Chapter 5 is fully deterministic. To account for the uncertainty inherent to strategic planning in general and to the volatile shipping market in particular, we conduct a sensitivity analysis and analyze the outcome of different scenarios. The use of stochastic models represents a logical direction for future research. It would allow to capture the uncertainty at the planning level already, to quantify the cost of different risks and to derive more robust solutions.

Further directions for future research are presented in the individual chapters.

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Chapter 2

Integrated Liner Shipping Network Design and Scheduling

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Abstract

In global liner shipping networks a large share of transported cargo is transshipped at least once between container vessels, and the total transportation time of these containers depends on how well the corresponding services are synchronized. We propose a problem formulation that integrates service scheduling into the liner shipping network design problem. Furthermore, the model incorporates many industry-relevant modeling aspects; it allows for leg-based sailing speed optimization, it is not limited to simple or butterfly-type services and it accounts for service level requirements like cargo transit time limits. The classic liner shipping network design problem is already a hard problem and in order to solve the extended version we propose a column generation matheuristic that uses advanced linear programming techniques. The proposed method solves LINER-LIB instances of up to 45 ports and finds new best solutions to all of them, outperforming existing methods reported in the literature. Additionally, we analyze the relevance of scheduling for liner shipping network design. The results indicate that neglecting scheduling and approximating transshipments instead may result in the design of liner shipping networks that underestimate cargo transit times and its implications.

2.1 Introduction

Containerization allows to seamlessly move cargo between different modes of transportation and is considered as one of the strongest drivers of international trade (Bernhofen et al., 2016). The backbone of the global trade network is formed by overseas cargo carriers that transport around 70% of the world seaborne trade in value (WTO, 2008). By offering comparatively very low transportation cost, container shipping networks have actually made today's global sourcing and production networks possible (Notteboom, 2012).

The largest carriers in the highly concentrated liner shipping market run several hundred vessels on more than a hundred *services*. Each service represents a sailing route that is regularly operated following a published schedule. Liner services are connected through common port calls that allow liner network operators to move cargo from one service to another. The movement of containers between services is called *transshipment* and enables large liner shipping companies to transport containers between almost any possible pair of ports around the globe.

The question of how to (re-)design a liner shipping network is at the core of optimization problems faced by cargo carriers and involves various tradeoffs. While shippers generally request economic, fast, and ideally direct port-to-port connections, shipping companies try to achieve high vessel capacity utilization and low operational costs. In order to achieve economies of scale, the average container vessel capacity has almost doubled between 2000 and 2014 and the capacity of the largest vessels has quadrupled since the Panamax era in the mid-1990s (Tran and Haasis, 2015a). On the downside, ultra-large container vessels (ULCV) can only call selected ports and therefore offer less direct port-to-port connections. Regional services operated by smaller vessels are commonly used to collect regional cargo and feed the ULCV at large hub ports. Indeed, the share of transshipped containers among global container port throughput has increased from 11% to 28% between 2008 and 2012 (Drewry, 2013). Today, in the network of Maersk Line, the world's largest overseas cargo carrier, more than half of all transported containers are transshipped at least once before arriving at their final destination.

A large share of transshipped containers requires liner service schedules to be well synchronized because they determine the transshipment times of cargo between services and, ultimately, the transit times of cargo. Moreover, the schedules determine the buffer times between port calls. With vessel delays being quite common, buffer times have a crucial impact on a carrier's reliability for both its schedule and cargo transit times. High transit time reliability and schedule reliability constitute a competitive advantage and justify higher transport rates (Notteboom, 2006; Vernimmen et al., 2007). Finally, the schedule of a liner shipping service implicitly defines the vessels' sailing speeds and thus the bunker consumption per sailing leg, which represents the largest operational cost for cargo carriers.

We present a mathematical model and solution method for the integrated Liner Shipping Network Design and Scheduling Problem (LSNDSP). Our model and solution method cover many practically relevant aspects, which to the best of our knowledge have never been combined into a single model. The proposed method finds solutions of high quality for LINER-LIB instances of up to 45 ports. To summarize, the contributions are:

- The liner shipping network design and scheduling problems are integrated into a single model.
- Transshipment times are modeled in an exact way and not approximated by a constant. Therefore, transit times are also exact.
- The method can efficiently handle service level requirements as, e.g., transit time limits for origin-destination demands, transit time dependent revenue functions or limits on the number of transshipments.
- The model and solution method allow for any type of liner service and are not limited to simple or butterfly type rotations.
- The solution method optimizes sailing speed on each sailing leg.
- We propose a novel column generation matheuristic that combines linear programming techniques with heuristics. The method can be used to construct new liner networks or to extend or improve existing liner networks.
- We report results for instances of the LINER-LIB benchmark suite involving up to 45 ports. We report new best solutions for all four considered LINER-LIB instances.
- The benefits of integrating scheduling with classic liner shipping network design are analyzed in detail.

The paper is organized as follows. Section 2.2 reviews related literature. In Section 2.3 we state and describe the LSNDSP. In Section 2.4 we first define a rich graph model (Section 2.4.1). Based on this model, we then state a mathematical formulation of the problem (Section 2.4.2) and shortly outline similarities and differences with other service network design models. In Section 2.5 we describe a column generation based solution algorithm and discuss relevant implementation details. These include a simple but highly efficient dual variable penalization technique and an improved solution algorithm for the time constrained multi-commodity flow problem. In Section 2.6 we present computational results based on the public LINER-LIB data instances. A comparison with results from the literature shows the effectiveness of the proposed solution method. Finally, concluding remarks and directions for future work are given in Section 2.7.

2.2 Literature review

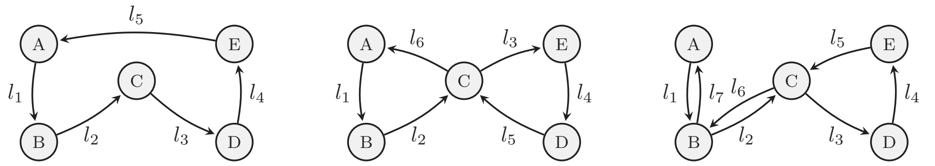
The LSNDSP contains many other studied maritime optimization problems as subproblems, among these the ship routing and scheduling problem, the fleet deployment problem, the speed optimization problem and the cargo flow problem. Researchers have addressed variations of these problems in different contexts and we refer the reader to the reviews by Christiansen et al. (2013), Meng et al. (2014) and Tran and Haasis (2015b) for an overview of optimization problems within liner shipping. In the following we focus on studies that address liner shipping *network* design problems that explicitly consider transshipments of containers between services.

The work by Agarwal and Ergun (2008) is regarded the first on liner shipping network design as it is defined and distinguished from other maritime routing problems. In particular, they were the first to consider a weekly service frequency and the possibility of cargo transshipments between services. Their model, however, does not account for transshipment costs. It is noteworthy that the model by Agarwal and Ergun (2008) also determines schedules for each service. It is a feature that most of the subsequent publications on liner shipping network design neglect. For better distinction from the LSNDSP we will use the acronym ULSNDP for unscheduled liner shipping network design problems in the remainder of the paper.

Álvarez (2009) presents a two-tier solution approach for the ULSNDP based on tabu search and column generation that is tested on a case study consisting of 120 ports. Gelareh and Pisinger (2011) propose a mixed integer programming formulation for regional liner shipping hub-and-spoke network design, together with a solution method based on Benders' decomposition. In order to promote research on liner shipping network design problems and to facilitate the comparison of solution approaches, Brouer et al. (2014a) develop a benchmark suite (LINER-LIB) and give an extensive introduction into the domain of liner shipping network design. They present a solution algorithm that is an extension of the heuristic by Álvarez (2009) and impose the requirement of (bi)weekly service frequencies. Another iterative heuristic for the ULSNDP was developed by Mulder and Dekker (2014) to solve a network design problem instance on the Asia-Europe trade lane. Brouer et al. (2014b) present a matheuristic to solve the ULSNDP. The proposed method is an improvement heuristic with an integer program at its core that is used to iteratively modify existing services and re-evaluate the network by solving cargo flow problems.

Brouer et al. (2015) and Karsten et al. (2017a) extend the work of Brouer et al. (2014b) by additionally considering restrictions on the total transit time and on the number of transshipments for each origin-destination container path. Karsten et al. (2017b) further extend the matheuristic by allowing varying speeds per leg on each service. The main contribution of this series of papers is the integration of *service levels* and thus of shippers' requirements and preferences into the liner shipping network design problem. Besides the pure transportation cost, the expected transit time and the number of transshipments of cargo are important decision criteria for shippers. Results are presented for all but the largest LINER-LIB instances, including large global liner networks. The service schedules are, however, considered to be independent and transshipment times are approximated by a constant of 48 hours.

Meng and Wang (2011) propose a mixed integer programming (MIP) formulation for a network design problem that additionally considers empty container repositioning to maintain a balance between container demand and supply at each port. Their solution method, however, does not design services, but selects services from a given set. Results are presented for randomly



(a) Simple service (b) Butterfly service (c) Complex service

Figure 2.1: Illustration of different service types

generated instances of 46 ports and a maximum of 60 shipping lines to select from.

Although heuristic solution approaches are dominant, a few exact algorithms for the ULSNDP have been proposed as well. Reinhardt and Pisinger (2012) present a compact formulation of the ULSNDP with transshipments, but services are not required to satisfy a weekly frequency. The model contains a large number of constraints, including *big-M* constraints, and the proposed branch-and-cut algorithm can solve to optimality only small instances with up to 10 ports and 12 origin-destination demands.

An alternative compact formulation of the ULSNDP is proposed by Plum et al. (2014) and applied to the two smallest instances of the LINER-LIB benchmark suite. An optimality gap remains for both instances. The main contribution is the model's capability to represent services of any type, i.e. services can visit any port multiple times. These types of services, which we denote *complex* in distinction to *simple* and *butterfly* type services (see Figure 2.1), are very common in practice.

Thun et al., 2016 analyze the potential cost savings that can be obtained by allowing complex services. Their study is motivated by the fact that the vast majority of models and solution methods is limited to simple and butterfly type services. Even though the considered instances are small (up to 7 ports and 14 demands), the results of their branch-and-price algorithm indicate that the use of complex services may allow for savings compared to networks solely consisting of simple and butterfly type services.

A stream of research related to the LSNDSP addresses models and solution methods for *service network design* in freight transportation. Note that in the service network design literature, *service* usually denotes a transportation service between e.g. terminals, whereas a service in liner shipping represents a whole sequence of port-to-port legs. Service network design problems are a generalization of network design problems and address tactical decision problems faced by freight carriers. Service network design problems are primarily characterized by their functionality rather than by the mode of transportation. The decisions that characterize service network design problems are the selection and scheduling of transportation services and the routing of cargo. Commonly, individual origin-destination demands are too low in service network design problems to be transported efficiently without consolidation. Crainic (2000) provides a classification of service network design problems and reviews modeling and solution approaches. The model we propose for the LSNDSP is a variation of a service network design model and shares similarities with scheduled cyclic service network design models as presented by Andersen et al. (2011) and Crainic et al. (2016). In Section 2.4 we will discuss differences between the LSNDSP and existing service network design problems and models in more detail.

2.3 Problem statement

The LSNDSP consists of two distinct but interdependent problems that are addressed simultaneously: On one hand, given an available fleet of vessels of different types, the decision maker has to define a set of services to be operated. On the other hand, the decision maker has to decide which origin-destination demand to transport and how to route it through the network. While the operation of services implies costs, the transportation of cargo generates revenues. Evidently, both problems are interdependent: the design and schedule of the network is driven by the potential revenues to gain, whereas the cargo routing options and transit times are defined by the network and schedule design.

A service represents a cyclic sequence of scheduled port calls and is operated by vessels of the same vessel class. The total round trip time of a service is equal to the sum of speed-dependent sailing times between served ports and a port stay time for each port call, covering loading and unloading operations. Figures 2.2a and 2.2b show two different Maersk Line services of total durations of 5 and 9 weeks, respectively. In the following we assume a weekly frequency for all services as predominant in the liner shipping industry. In order to ensure a weekly service frequency, the number of vessels deployed on a service has to be equal to the duration of the service in weeks. The maximum duration of a service is limited by the number of available vessels per vessel class.

A vessel class is defined by its capacity in forty-foot equivalent (FEU) units, a minimum and maximum speed and by its ship measurements. A vessel's dimension or draft may disqualify the vessel class for some ports or canals and a vessel class' speed limitations may not allow it to operate a particular schedule. The cost of operating a vessel of a particular class consists of a (weekly) charter rate and sailing speed dependent fuel cost.

A demand represents a weekly quantity of FEU units for a particular origin-destination pair of ports. Each demand is associated with a unit revenue. Additionally, transit time limits may apply, reflecting a demand's time-sensitivity. Demands of different revenues and time-sensitivities may exist for the same origin-destination pairs. Cargo can generally be transshipped between services, but every transshipment implies additional costs for cargo handling at the transshipment port. The transshipment time depends on the schedules of the unloading and the loading service. If a minimum transshipment time is not met, cargo may have to wait one week for the next vessel to arrive. A limit on the number of transshipments may be defined for

(a) Maersk Line Meso American Express round trip

(b) Maersk Line Amazonia Caribbean Express round trip

Figure 2.2: Two examples of Maersk Line services (Maersk Line, 2017). The ports of Cartagena, Colombia, and Manzanillo, Panama, are called by both services and allow cargo to be transshipped between the two services. Note that both services are complex services.

a demand, reflecting the shippers preference towards direct shipping routes. A demand can, but does not have to be served by the cargo carrier. Moreover, it can be fulfilled partially.

The objective is to design and schedule a network of services such that the (weekly) cost of operating the services minus the earned revenue for serving demands is minimized. The cargo carrier cannot use more than the available number of vessels for each vessel class and has to satisfy transit time or transshipment restrictions for all served demands.

2.4 A model for the LSNDSP

The model we propose for the LSNDSP is formulated over a directed time-space graph. We first describe the graph modeling idea and define resulting graphs in Section 2.4.1. The mathematical model of the LSNDSP is presented in Section 2.4.2.

2.4.1 Graphs

We model the problem over a directed time-space graph, which allows to not only model varying sailing speeds between subsequent port calls, but also schedule dependent transshipment times between different services that call the same port. The (weekly) time is discretized into uniform steps of length $\bar{h}$ (a divisor of 168 hours), represented by the set $H = \{0, 1 \cdot \bar{h}, 2 \cdot \bar{h}, \ldots, 168 - \bar{h}\}$. Let P denote the set of ports and L the set of legs, representing all feasible port-to-port sailings. Set L may be larger than $|P| \times |P|$ because for some pairs of ports different sailing routes may exist (e.g. one using Suez canal and another one sailing around Africa).

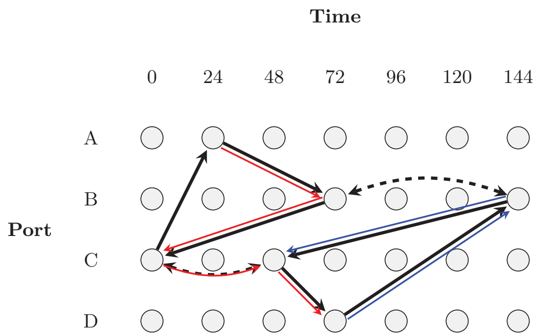


Figure 2.3: Illustration of a time-space graph G. Solid arcs represent sailings between ports and dashed arcs represent transshipments. The colored arcs denote examples of cargo paths; the blue path from port D to port C is direct, whereas the red path from port A to port D requires a transshipment at port C. Backward arcs represent sailings or transshipments that end in the following week.

All feasible services and all feasible cargo paths can be represented in a basic directed graph, denoted $G = (V, A)$, where V and A are its vertex set and arc set, respectively. Set V denotes ports at a particular departure time and contains $|P| \times |H|$ vertices, namely, one vertex for each port $i \in P$ and each time $h \in H$. A vertex $(i, h) \in V$ represents a departure from port i at time h. Figure 2.3 illustrates a small time-space graph with two services (each with a one-week duration) and two cargo paths.

The arc set A connects some of the vertices to each other and is the union of two arc sets A^S and A^T. Arc set A^S represents *sailing arcs* and connects two port-time vertices (i_1, h_1) and (i_2, h_2) with $i_1 \neq i_2$ to each other, if a vessel class exists that can sail a leg $l \in L$ from port i_1 to port i_2 departing at time h_1 such that it is ready for departure from port i_2 again at time h_2. The duration t_a of an arc $a \in A^S$ includes the sailing time between ports (possibly including waiting time outside of the destination port) and the port stay time at the destination port. Note that the one-week time horizon is cyclic over a cylinder, which means that all times should be computed modulo 7 days (168 hours). Let us, for example, consider a sailing leg from port i_1 to port i_2 departing at time $h_1 = 8$: If the minimum and maximum sailing times between ports i_1 and i_2 are 128 and 152 (all times in hours), respectively, the port stay time is 24 and a time discretization of $\bar{h} = 8$ is used, the earliest weekly departure time from port i_2 after this leg is $(8 + 128 + 24) \mod 168 = 160$ and the latest weekly departure time is $(8 + 152 + 24) \mod 168 = 16$. Hence, the corresponding sailing arcs would connect vertex $(i_1, 8)$ to vertices $(i_2, 160)$, $(i_2, 0)$, $(i_2, 8)$ and $(i_2, 16)$. The solid arcs in Figure 2.3 represent selected sailing arcs, including two arcs representing the same sailing (between ports B and C) but at a different speed.

The second subset of arcs, *transshipment arcs* A^T, represents the transshipment of cargo from one service to another at a port $i \in P$ with a particular layover time. Transshipment arcs exist between all port-time vertices (i_1, h_1) and (i_2, h_2) with $i_1 = i_2$. The dashed arcs in Figure 2.3 illustrate transshipments. The duration t_a associated with a transshipment arc $a \in A^T$ depends on the minimum layover time necessary for a transshipment at a particular port. Note that the layover time is defined as the time between arrival of the unloading vessel and the departure of the loading vessel. Hence, given a port stay time of 24 and a minimum layover time of 48, the resulting outgoing transshipment arcs of e.g. vertex $(i_1, 8)$ are of duration $48 - 24 = 24$ to $160 + 48 - 24 = 184$ and thus connect to all other vertices of the same port, $(i_1, 32), (i_1, 40), \ldots, (i_1, 160), (i_1, 0), \ldots, (i_1, 24)$, in the order of increasing duration.

In theory, there can be $|P| \times |P - 1| \times |H|^2$ arcs between vertices $v, u \in V$. This number is even larger if parallel arcs exist between port-time vertices due to the same sailing at different speeds arriving a multiple of weeks apart (i.e. same sailing (i_1, h_1) to (i_2, h_2), but different arc durations t_a due to the weekly cylinder). These cases may occur, as e.g. a sailing between Europe and the U.S. east coast can take a week or two, depending on the sailing speed. In order to not overload notation, we will denote in the remainder of the paper an arc $a = (u, v)$ by its origin and destination vertices u and v, assuming a unique duration t_a for every pair (u, v). Our implementation and computational tests, however, account for parallel arcs of different durations. Graph G contains all vertices and arcs that are feasible for at least one vessel class and cargo path.

Different port restrictions as well as varying vessel-dependent feasible ranges of speeds make different vertices and arcs of graph G feasible for each vessel class. We denote the set of vessel classes by E. Let $G_e = (V_e, A_e)$ denote the subgraph that contains all vertices and arcs feasible for vessel class $e \in E$. Set $V_e \subseteq V$ contains all vertices corresponding to time-copies of ports that can be visited by a vessel in class $e \in E$. Set $A_e^S \subseteq A^S$ contains all sailing arcs that are feasible for a vessel in class $e \in E$, where feasibility of an arc $a = (v, u)$ depends on whether $v, u \in V_e$ and whether the sailing time associated with arc a is feasible for this vessel class. Note that arc set A_e does not contain any transshipment arcs. Sets $A_e^+(v)$ and $A_e^-(v)$ are used to denote outgoing and incoming arcs of vertex $v \in V$ that are feasible for vessel class $e \in E$. We use the notation $E_v \subseteq E$ and $E_a \subseteq E$ to denote the vessel classes that are allowed to use vertices $v \in V$ and arcs $a \in A^S$, respectively.

Different vertices and arcs may be feasible for different demands, which we denote by K. Feasible paths for a demand $k \in K$ with origin O_k and destination D_k correspond to paths in G that start at a vertex $v \in V$ whose associated port is equal to O_k and end at a vertex $v \in V$ whose associated port corresponds to D_k. We define subgraphs $G_k = (V_k, A_k)$ for each demand $k \in K$ that contain all these paths. Set $V_k \subseteq V$ contains all vertices $v \in V$ whose associated port can be part of a feasible path from O_k to D_k of demand k while respecting the demand maximum transit time. Set $A_k \subseteq A$ contains all arcs in A linking vertices in V_k, except all arcs that would be infeasible with respect to the maximum transit time, and all arcs associated with a sailing ending in O_k or starting in D_k. Accordingly, we use the notation $K_v \subseteq K$ and $K_a \subseteq K$ to denote the demands that are allowed to use vertices $v \in V$ and arcs $a \in A$, respectively.

2.4.2 Mathematical model

In this section, we provide a MIP formulation for the LSNDSP. The LSNDSP is formulated on graph $G = (V, A)$. The goal of the model is to design services and to select cargo paths that use the services while minimizing total cost minus total revenue.

The binary decision variables y_a^e indicate whether a sailing leg represented by arc $a \in A^S$ is sailed by vessel class $e \in E$ ($y_a^e = 1$) or not ($y_a^e = 0$). The set Ω contains all feasible cargo paths; it can be split into disjoint sets Ω_k, one for each demand $k \in K$. The continuous decision variables x_j^k denote the number of containers of demand $k \in K$ that are transported along path $j \in \Omega_k$. While the number of transported containers can only be integer, the error resulting from relaxing the integrality requirements on these decision variables is negligible in practice given that vessels can carry several thousand containers (Brouer et al., 2011).

The binary parameters b_{ja} are equal to 1, if container path $j \in \Omega_k, k \in K$ uses arc $a \in A^S$ and 0 otherwise. Parameters U^e and N^e denote the capacity and the available number of vessels of a vessel class $e \in E$, respectively. The cost parameter c_a^e represents the cost of vessel class $e \in E$ sailing arc $a \in A^S$. It includes the vessel time charter rate proportional to the duration of the sailing, fuel cost for sailing and idling proportional to the sailing speed and idling time at port, port call fees, and canal transit fees (if applicable). The cost parameter c_j^k denotes the unit net cost of transporting a container of demand $k \in K$ along path $j \in \Omega_k$. This net cost is computed as the unit loading cost at origin port O_k, the unit unloading cost at destination port D_k, the transshipment cost along arcs $a \in A^T$ minus the unit revenue z^k. Parameter q_k denotes the number of containers associated with demand $k \in K$ and p^k represents a demand rejection penalty. Note that adding such a penalty in the objective function is equivalent to increasing the revenue of a demand plus adding a constant term to the objective function value.

The LSNDSP can be formulated as the following MIP model.

$$\min \quad \sum_{a \in A^S} \sum_{e \in E_a} c_a^e y_a^e + \sum_{k \in K} \sum_{j \in \Omega^k} c_j^k x_j^k + \sum_{k \in K} \left(q^k - \sum_{j \in \Omega^k} x_j^k \right) p^k \tag{2.1}$$

$$\text{s.t.} \qquad \sum_{j \in \Omega^k} x_j^k \leq q^k, \qquad \forall k \in K \qquad [\alpha] \qquad (2.2)$$

$$\sum_{e \in E} \sum_{a \in A_e^+(v)} y_a^e \leq 1 \qquad \forall v \in V \qquad [\beta] \qquad (2.3)$$

$$\sum_{a \in A_e^+(v)} y_a^e - \sum_{a \in A_e^-(v)} y_a^e = 0 \qquad \forall e \in E, v \in V^e, \qquad [\gamma] \qquad (2.4)$$

$$\sum_{k \in K_a} \sum_{j \in \Omega^k} b_{ja} x_j^k - \sum_{e \in E_a} U^e y_a^e \leq 0 \qquad \forall a \in A^S \qquad [\lambda] \qquad (2.5)$$

$$\sum_{a \in A^e} t_a y_a^e \leq 168 N^e \quad \forall e \in E \qquad [\pi] \qquad (2.6)$$

$$x_j^k \geq 0 \qquad \forall k \in K, j \in \Omega^k \qquad (2.7)$$

$$y_a^e \in \{0, 1\} \qquad \forall a \in A^S, e \in E_a. \qquad (2.8)$$

The objective function (2.1) minimizes the total net cost that is computed as the sum of the service costs and the cargo-related costs (unloading, loading, and transshipment costs plus a rejection penalty) minus the total revenue for the transported demands. Constraints (2.2) limit the number of containers that can be transported for each demand. Constraints (2.3) ensure that no two services depart from the same port at the same time. Constraints (2.4) are flow conservation constraints for the service variables. For every vessel class they require that the flow into a vertex be equal to the flow out of this vertex, i.e. if a service arrives at a port at a particular time, it has to leave it again after the port stay time. Constraints (2.5) link the service variables to the cargo paths. They ensure that if cargo is transported from one port to another at a particular departure time and with a particular speed (sailing time), then a vessel is assigned to this sailing with a sufficient capacity. Fleet availability is enforced by constraints (2.6). Finally, nonnegativity and binary requirements (2.7)–(2.8) restrict the domains of the variables.

The following trivial valid inequalities, known as *strong linking inequalities* (Frangioni and Gendron, 2009), can be used to tighten the linear programming (LP) relaxation of (2.1)–(2.8):

$$\sum_{j \in \Omega^k} b_{ja} x_j^k - \sum_{e \in E_a} \min\{q^k, U^e\} y_a^e \leq 0, \quad \forall k \in K, a \in A^S \quad [\phi] \qquad (2.9)$$

For practical applications the total number of valid inequalities (2.9), $O(|A^S| \times |K|)$, is usually too large to consider all of them explicitly.

If the binary restrictions on variables y_a^e are relaxed, the model becomes a linear program. Below we will use $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, $\boldsymbol{\lambda}$, $\boldsymbol{\pi}$ and $\boldsymbol{\phi}$ to denote the dual variable vectors corresponding to constraints (2.2)-(2.6) and valid inequalities (2.9) of the LP relaxation of (2.1)–(2.9).

Model (2.1)–(2.8) is a variation of a service network design model (see e.g. Crainic, 2000). More specifically, the LSNDSP is a cyclic, scheduled network design problem and a generalization of the problem described by Andersen et al. (2011) or Crainic et al. (2016). Below we discuss some of the differences. In Andersen et al. (2011) and Crainic et al. (2016), the fleet of vehicles is assumed to be homogeneous and required to return to their origin within the schedule length (one week); in the LSNDSP, different vessel classes exist and the duration of a liner shipping service is only limited by the available number of vessels of a particular vessel class. Andersen et al. (2011) do not consider transshipments of commodities; Crainic et al. (2016) allow for transshipments, but no cost is imposed. Although it is a minor difference at first glance, a positive cost for transshipments makes the problem more difficult, because it generally increases the LP gap for service network design instances; transshipments occur less often when solving the LP relaxation, as fractions of services can be used to offer direct transportation for any demand and therefore, higher transshipment costs will increase the upper bound more than they will lift the lower bound. In many service network design applications, including the one by Andersen et al. (2011), all or some of the origin-destination demands *have* to be satisfied fully or partially. The resulting lower bounds on the transported demand reduce the solution space and allow to use additional sets of valid inequalities, as for example *cutset* inequalities (Magnanti et al., 1995). On the other hand, as Andersen et al. (2011) point out, lower bounds on transported demand may make it more difficult to find a feasible solution. Last but not least, origin-destination demands are only associated with a port in case of the LSNDSP, whereas in Andersen et al. (2011) they are additionally required to be served at a particular time. Again it results in a reduction of the solution space, but may increase the difficulty of finding a feasible solution.

We show that the LSNDSP is $\mathcal{NP}$-hard in the strong sense by reduction from the asymmetric traveling salesman problem (ATSP).

Proposition 1. *The LSNDSP is $\mathcal{NP}$-hard in the strong sense.*

Proof. Let $\bar{h} = 168$ and thus $|H| = 1$. The time space graph $G(V, A)$ contains exactly one vertex per port $p \in \{1, \dots, |P|\}$ in this case and arc set A^T is therefore empty. Let vertex set V and arc set A correspond to the vertex and arc set of the ATSP. Assume a single vessel class $e \in E$ ($|E| = 1$) with capacity $U^e \geq |P| - 1$, an infinite number of available vessels $N^e = \infty$ and let arc costs c_a^e of the single vessel class e correspond to the arc costs in the ATSP. For every port $\hat{p} \in P, \hat{p} \neq 1$ we define exactly one demand $k \in K$ with port $p = 1$ as origin and port $p = \hat{p}$ as destination. Let quantity $q^k = 1$ and penalty $p^k = \infty$ for all demands $k \in K$. By solving the LSNDSP, we obtain a minimum-cost service that visits each port (i.e. vertex) exactly once in order to serve all demands. The service corresponds to a minimum-cost Hamiltonian cycle on directed graph $G(V, A)$ and is an optimal solution to the corresponding instance of the ATSP. We can thus solve the ATSP by solving a LSNDSP. $\square$

2.5 A column generation matheuristic

To solve model (2.1)–(2.8), we propose a column generation matheuristic. The general algorithm design is motivated by the following observations: First, the linear relaxation of capacitated network design problems is known to be notoriously weak and no efficient exact methods exist for large instances of the general problem. Second, heuristics that do not make use of mathematical programming techniques are usually also not very successful for capacitated network design problems, as Crainic (2000) concludes.

The method we propose makes use of the efficiency of heuristics and uses LP techniques to guide the search. Algorithm 1 gives a high-level description of the column generation matheuristic (LSNDSP-CGM). Before explaining each step of the algorithm in detail, we introduce some additional notation. We use χ and X to denote a single feasible integer solution and a set of feasible integer solutions of model (2.1)–(2.9), respectively. A feasible integer solution χ is defined by a set of services S_χ and a set of cargo paths Ω_χ. A service $s \in S_\chi$ is defined by a vessel class $e(s)$ and a set of arcs A^S_s that represents the route and schedule operated by service s.

Algorithm 1 LSNDSP-CGM

Require: Initial (possibly empty) solution χ defined by cargo paths Ω_χ and services S_χ
Require: Parameters $T^{\max}$, $I^{\max}_{\text{crg}}$, $N^{\max}_{\text{crg}}$, F_{MIP}, F_{Pol}

Initialize $\Omega' \leftarrow \Omega_\chi$, $\Lambda' \leftarrow \Lambda(\Omega')$, $\Phi' \leftarrow \emptyset$, $\mathcal{T} \leftarrow \emptyset$, $E' \leftarrow \emptyset$, $S' \leftarrow S_\chi$
Build model $M = M(\Omega', \Lambda', \Phi', \mathcal{T}, E', S')$
Set best known solution $\chi^* \leftarrow \chi$
while (*runtime* $< T^{\max}$) **do**
  $E', S' \leftarrow$ FixPartOfNetwork(M, χ^*) ▷ Section 2.5.1
  $\Omega', \Lambda', \Phi' \leftarrow$ColRowGen($M$) ▷ Sections 2.5.2 to 2.5.4
  $X \leftarrow$GenFeasibleSols(M) ▷ Section 2.5.5
  for $\chi \in X$ **do**
    $\chi \leftarrow$CargoAllocation(χ) ▷ Section 2.5.6
  end for
  $\chi' = \arg\min_{\chi \in X} z(\chi)$
  if $z(\chi') < z(\chi^*)$ **then**
    $\chi^* \leftarrow \chi'$
    Delete all cargo paths: $\Omega' \leftarrow \emptyset$
    Delete all tabu constraints: $\mathcal{T} \leftarrow \emptyset$
  else
    Add tabu constraints for solution χ': $\mathcal{T} \leftarrow \mathcal{T} \cup \mathcal{T}(\chi')$ ▷ Section 2.5.7
  end if
end while

Let $M(\Omega', \Lambda', \Phi', \mathcal{T}, E', S')$ denote a restricted version of model (2.1)–(2.9) that is dynamically updated during the execution of method LSNDSP-CGM. Ω', Λ' and Φ' denote the subsets of cargo path variables, arc capacity constraints (2.5) and strong linking inequalities (2.9), respectively, that are part of the model. Set $\mathcal{T}$ represents tabu constraints that the method LSNDSP-CGM may add to the model. The tabu constraints are described in detail in Section 2.5.7. Similarly, sets $E' \subset E$ and $S' \subseteq S$ represent subsets of fixed vessel classes and fixed services, respectively, that define variable fixing constraints. The fixing procedure and resulting variable fixing constraints are described in Section 2.5.1. For ease of reading we use the shorter notation M as equivalent to $M(\Omega', \Lambda', \Phi', \mathcal{T}, E', S')$.

Method LSNDSP-CGM requires an initial solution, which, however, can also be an empty liner shipping network, and a set of input parameters. The algorithm starts with initializing the sets that define model $M(\Omega', \Lambda', \Phi', \mathcal{T}, E', S')$ (line 1). The initial set of cargo paths Ω' contains all cargo paths of the initial solution, if any. The initial set of arc capacity constraints Λ' depends on the initial set of cargo paths and contains constraints for all arcs that are used by at least one cargo path. Formally, $\Lambda(\Omega')$ contains arc capacity constraints for the set of arcs $\{a \in A^S : \sum_{j\in\Omega'} b_{ja} > 0\}$. All other sets Φ', $\mathcal{T}$, E' and S' are initially empty. Model $M(\Omega', \Lambda', \Phi', \mathcal{T}, E', S')$ is generated based on the initial sets (line 2). Note that any update of the model-defining sets implicitly updates the model as well.

The core of the algorithm is an iterative destroy-and-(re-)build mechanism described by lines 4 to 19. We provide a high-level overview of the algorithm core first and discuss its key components in detail in subsequent subsections.

At the beginning of each iteration, the current network is partially fixed (line 5). Fixed services cannot be modified and fixed vessel classes cannot be used to design new services during an iteration. The main rationale behind the partial fixing of the network is to keep the optimization problem tractable. The fixing procedure is discussed in detail in Section 2.5.1.

The column and row generation (CRG) phase is a crucial component of the LSNDSP-CGM. The CRG procedure is based on the LP relaxation of model M, denoted LP(M), and aims at generating new cargo paths (line 6). We summarize the CRG algorithm in Section 2.5.2 and discuss the pricing problems together with an efficient label setting algorithm in Section 2.5.3. In Section 2.5.4 we present an effective dual penalization procedure used by the CRG algorithm. The dual penalties play a key role during the CRG phase, particularly for the pricing of new cargo paths.

After the generation of cargo paths, the method generates multiple feasible integer solutions. Emphasis lies on the number and diversification of feasible networks rather than on the optimal network for the given cargo paths Ω'. The generation of feasible solutions consists of two steps: First, a MIP solver is run and a set of feasible solutions X is collected. Second, the MIP polishing heuristic of Rothberg (2007) is run to enlarge and diversify the set X (line 7). For details see Section 2.5.5.

Each single feasible solution $\chi \in X$ previously generated is based on the subset Ω' of cargo paths. In lines 8 to 10 the algorithm solves the Cargo Allocation Problem (CAP) for each feasible solution in order to find *all* negative reduced cost cargo paths for the fixed network corresponding to each solution χ. The CAP is described in Section 2.5.6. The resulting set of solutions X represents a set of networks and each network's optimal cargo allocation.

If the best solution χ' among all solutions X constitutes a new global best solution, all cargo paths and tabu constraints are removed from model M (lines 14 to 15). Otherwise, tabu constraints $\mathcal{T}(\chi')$ that forbid solution χ' in subsequent iterations are added to increase diversification (line 17). The tabu constraints are described in more detail in Section 2.5.7.

2.5.1 Partial fixing of network

Each iteration of algorithm LSNDSP-CGM starts with fixing a part of the network of the current best solution. Vessel classes and services may be fixed for both practical and algorithmic reasons. Practical reasons will be given by the particular decision problem that is addressed. The decision maker may, for example, only want to re-design feeder services or introduce new services in a particular region without modifying the existing network.

From an algorithmic point of view, fixing vessel classes and services trivially reduces the complexity of the optimization problem at each iteration. For the particular optimization problem, which is a variation of a capacitated network design problem, it also helps to generate cargo paths of higher quality during the column-and-row-generation phase. We will elaborate this argument further in Section 2.5.2.

At each iteration, algorithm LSNDSP-CGM fixes all but one vessel classes, i.e. $|E'| = |E| - 1$. The set $E \setminus E'$, containing a single non-fixed vessel class, determines the vessel class for which new services can be generated during an iteration. The selection is random, except for the first $|E|$ iterations; in the first iteration of LSNDSP-CGM all but the largest vessel class are fixed, in the second iteration all but the second largest vessel class are fixed and so on. This deterministic fixing rule during the first $|E|$ iterations aims at preventing small vessels to be initially deployed on the trades with the largest volumes, particularly when the algorithm is used to build a network from scratch. The set of fixed services $S' \subseteq S_{\chi^*}$ is chosen to contain all services of all fixed vessel classes. Consequently, all services of the non-fixed vessel class are destroyed and rebuilt during an iteration of LSNDSP-CGM.

Let $e(s)$ denote the vessel class that is deployed on service $s \in S$. The variable fixing constraints resulting from the choice of S' are

$$y_a^{e(s)} = 1, \quad \forall a \in A_s^S, s \in S' \tag{2.10}$$

For fixed vessel classes $e \in E'$, all corresponding decision variables y_a^e are fixed to zero if not part of a fixed service $s \in S'$.

$$y^e_a = 0, \quad \forall e \in E', a \in \{A^S_e \setminus \cup_{s \in S'} A^S_s\} \tag{2.11}$$

We have tested various other strategies for both fixing vessel classes and services, including less aggressive strategies that allow services to be generated for more than one vessel class in each iteration. However, none of these strategies outperformed the simple one described above.

2.5.2 Column and row generation

During the CRG phase the integrality constraints (2.8) are relaxed and the resulting model becomes a linear program. We call the relaxed version of model $M(\Omega', \Lambda', \Phi', \mathcal{T}, E', S')$ the *restricted master problem* (RMP). The CRG phase aims at generating good cargo paths, rather than solving the RMP to optimality; i.e. cargo paths that are likely to be part of a feasible *integer* solution.

This motivates the following CRG algorithm design (see Algorithm 2): At each CRG iteration we first solve the RMP to obtain updated dual variable values (line 3). A set Ω'' of cargo paths is generated by solving pricing problems (line 5). The pricing problems and a label setting algorithm to solve them are described in detail in Section 2.5.3. The $N^{\max}_{\text{crg}}$ most negative reduced cost paths per demand are added to the current set of columns Ω' (line 6). Based on the newly added cargo paths, the current set of arc capacity constraints Λ' is updated by adding missing constraints (2.5) for all arcs that are used by at least one cargo path $j \in \Omega''$ (line 7). The updated RMP is solved (line 8) and all violated strong linking inequalities are identified and added to the RMP (line 9).

Besides quickly increasing the number of constraints in the RMP, the rather aggressive strategy of adding all violated strong linking inequalities also comes at the expense of an extra call to

Algorithm 2

1: **procedure** ColRowGen(M)
2:     **for** $i \in \{1, \dots, I^{\max}_{\text{crg}}\}$ **do**
3:         Solve RMP
4:         $\tilde{\boldsymbol{\lambda}} \leftarrow$ UpdateDualPenalties(E') ▷ Section 2.5.4
5:         Generate negative reduced cost cargo paths Ω'', at most $N^{\max}_{\text{crg}}$ per demand ▷ Section 2.5.3
6:         Add generated cargo paths Ω'' to RMP: $\Omega' \leftarrow \Omega' \cup \Omega''$
7:         Add arc capacity constraints: $\Lambda' \leftarrow \Lambda' \cup \Lambda(\Omega'')$
8:         Solve RMP
9:         Identify and add violated strong linking inequalities Φ'': $\Phi' \leftarrow \Phi' \cup \Phi''$
10:     **end for**
11: **end procedure**

the LP solver at each CRG iteration and increases the number of pricing problems to solve (discussed in Section 2.5.3). However, preliminary experiments have shown that the gain in solution quality significantly outweighs the increased running time per CRG iteration.

At each call of the CRG procedure at most $I_{\text{crg}}^{\max}$ column and row generation iterations are performed.

2.5.3 A label setting algorithm for the cargo path pricing problems

There is a pricing problem for each demand $k \in K$. For a demand k, the pricing problem can be formulated as a resource-constrained shortest path problem defined on a graph $\bar{G}_k = (\bar{V}_k, \bar{A}_k)$. Vertex set $\bar{V}_k = V_k \cup \{o_k, d_k\}$, where o_k and d_k are the source and sink vertices. Arc set $\bar{A}_k = A_k \cup A_k^O \cup A_k^D$, where A_k^O contains arcs linking the source vertex o_k to every vertex in V_k associated with port O_k, and A_k^D contains arcs linking every vertex in V_k associated with port D_k to the sink vertex d_k. With each arc $a \in \bar{A}_k$, we associate a cost c_a, a duration t_a and a reduced cost $\bar{c}_a$. We define cost c_a of an arc $a \in \bar{A}^k$ such that it equals the loading and unloading cost of a container on arcs $a \in A_k^O$ and $a \in A_k^D$, respectively, and the transshipment cost of a container on arcs $A_k \cap A^T$. For sailing arcs, c_a is zero. The reduced cost of an arc $a \in \bar{A}_k$ depends on the actual arc cost c_a, non-positive dual variable vectors $\boldsymbol{\lambda}$ and $\boldsymbol{\phi}$ and a non-negative dual penalty vector $\tilde{\boldsymbol{\lambda}}$ that is defined over all sailing arcs (and described in detail in Section 2.5.4):

$$\bar{c}_a := -\lambda_a - \phi_{ak} + \tilde{\lambda}_a \quad \forall a \in \bar{A}_k \cap A^S \tag{2.12a}$$

$$\bar{c}_a := c_a \quad \forall a \in \bar{A}_k \setminus A^S \tag{2.12b}$$

The reduced cost of a cargo path is defined as $\bar{c}_j^k := -z_k - \alpha_k + \sum_{a \in \bar{A}_k} b_{ja} \bar{c}_a$. Note that all reduced arc costs $\bar{c}_a$ are non-negative.

The duration t_a associated with each arc is defined as follows: For any sailing or transshipment arc $a \in \bar{A}_k \cap A$ the duration equals the sailing and transshipment time, respectively, as defined for graph G_k. All arcs $a \in A_k^O$ and $a \in A_k^D$ have an associated duration of $t_a = 0$.

The pricing problem consists of finding in $\bar{G}_k$ a negative reduced cost shortest path between o_k and d_k that does not exceed the maximum transit time T_k. A labeling algorithm can be used to solve this problem (see e.g. Irnich and Desaulniers, 2005). We present a label setting algorithm similar to the one described by Karsten et al. (2015). Different to Karsten et al. (2015), we cannot solve one aggregate subproblem per origin port, because the reduced cost of arcs may differ for different demands $k \in K$ due to the dual variable values ϕ_{ak} (see definition (2.12a)). Below we describe the label setting algorithm for solving one pricing problem per demand.

A label has two components (C, T). The C component stores the reduced cost of the path associated with the label and the T component represents the cumulative transit time along this path. Resource windows $[0, \bar{C}_{kv}]$ and $[0, \bar{T}_{kv}]$ are associated with each vertex $v \in \bar{V}_k$ in order to impose bounds on the cost and time components of each label. Different to Karsten et al. (2015) we do not leave the cost component unconstrained and use tighter resource windows for the time component. A valid upper bound on the cost component of a label can be derived from the requirement that the reduced cost has to be negative; it follows from $-z_k - \alpha_k + \sum_{a \in \bar{A}_k} b_{ja}\bar{c}_a < 0$ that the cost of a label cannot exceed $z_k + \alpha_k$ and thus $\bar{C}_{kv} := z_k + \alpha_k$ is valid for any vertex v (recall that $\bar{c}_a \geq 0$ for all arcs $a \in \bar{A}_k$). Similarly, the duration of a label cannot exceed the demand's maximum transit time and thus $\bar{T}_{kv} := T_k$ is valid for any vertex v. The upper bounds $\bar{C}_{kv}$ and $\bar{T}_{kv}$ are the tightest possible upper bounds for those vertices $v \in \bar{V}_k$ that represent the destination port of demand $k \in K$. Tighter upper bounds $\bar{C}_{ku}$ and $\bar{T}_{ku}$ can be derived recursively for vertices $u \in \bar{V}_k$ by finding the tightest upper bounds that satisfy $\bar{C}_{ku} + \bar{c}_a \geq \bar{C}_{kv}$ and $\bar{T}_{ku} + t_a \geq \bar{T}_{kv}$ for all $a = (u, v) \in A$, respectively. We only apply the recursive tightening of upper bounds for the time component, because these bounds have to be calculated only once at initialization and, unlike the upper bounds on the cost component, do not have to be updated. This preprocessing technique can be generalized towards one-to-all shortest path problems.

Standard label extension functions and a standard dominance rule are used. Labels are treated in the order of increasing path durations. The time discretization allows us to cluster the set of unprocessed labels by discretized durations. It makes any explicit sorting of labels unnecessary and therefore saves further computational time.

2.5.4 Dual penalties

The reduced cost of any cargo path $j \in \Omega_k, k \in K$ consists of a demand dependent term and the reduced cost of arcs that constitute the path. Every path by definition contains at least one sailing arc and the reduced cost of a sailing arc $a \in A^S$, as defined by equation (2.12a), only depends on the dual variables λ_a and ϕ_{ak} if the dual penalty $\tilde{\lambda}_a$ is zero. Both dual variables λ_a and ϕ_{ak} are zero for all arcs that are not used by any cargo path $j \in \Omega'_k, k \in K$ in the RMP, because the corresponding constraints are relaxed or, if not relaxed, non-binding in that case. Consequently, the reduced cost $\bar{c}_a$ of any sailing arc $a \in A^S$ is zero if $\sum_{k \in K} \sum_{j \in \Omega'_k} b_{ja} = 0$. For any given solution to the RMP, we will call such an arc *unused* if additionally no capacity is deployed on that same arc ($\sum_{e \in E} y^e_a = 0$).

Together with the fact that $\bar{c}_a$ is always non-negative, we can make the following two observations:

1. Two unused sailing arcs have the same reduced cost of zero, independent of the sailing distance and speed that they represent.

2. The reduced cost of an unused sailing arc is always lower or equal to the reduced cost of an arc with any deployed capacity.

For the labeling algorithm described above it has quite counter-intuitive implications: Consider a network and corresponding graph of three ports A, B and C (omitting time for simplicity).

Assume one fixed liner service that sails $A \to B \to C \to A$. If sets $\Omega'_k, k \in K$ are initially empty, both cargo paths $B \to A$ and $B \to C \to A$ have the same reduced cost of zero. As direct path $B \to A$ is of shorter duration, it will dominate the longer path $B \to C \to A$ despite the fact that an empty vessel is deployed on $B \to C \to A$ and no capacity is deployed on $B \to A$.

Intuitively, the reduced cost of path $B \to A$ is underestimated. We present a simple, but effective dual variable penalization procedure to overcome this issue. We define a set of dual penalty values $\tilde{\lambda}_a$ defined over the set of sailing arcs $a \in A^S$. The similarity in notation to the dual variables λ_a is intended and the penalty values can in fact be seen as penalties of the zero-valued λ_a dual variable values for *unused* arcs.

Formally, we define the dual penalties as

$$\tilde{\lambda}_a := \begin{cases} \eta \cdot \min\limits_{e \in E_a \setminus E'} \{c_a^e / U^e\}, & \text{if } \sum\limits_{k \in K} \sum\limits_{j \in \Omega'_k} b_{ja} = 0 \wedge \sum\limits_{e \in E} y_a^e = 0 \\ \\ 0, & \text{otherwise} \end{cases} \qquad \forall a \in A^S \tag{2.13}$$

η is a non-negative parameter which we call the *dual penalty multiplier*. The penalties $\tilde{\lambda}_a$ only take a non-zero value, if the corresponding arc is unused at the time of updating the dual penalties and if the dual penalty multiplier η is not zero. The term c_a^e/U^e represents the marginal cost of sending one more container on a vessel of class $e \in E$ on arc $a \in A^S$. The marginal cost c_a^e/U^e is relative to the distance and sailing speed associated with arc a and to the fuel consumption function of vessel class e. The penalty is set to the lowest value of all feasible vessel classes $E \setminus E'$. The penalties are updated each time before the cargo path pricing problems are solved (see Algorithm 2).

Whereas the penalty values are defined per arc, dual penalty multiplier η is a global parameter used to control the overall impact of penalties $\tilde{\lambda}$. As the penalties may increase the reduced cost of cargo paths, they may in fact cause column generation to stop prematurely; the dual penalty multiplier η can be used to lower the penalty values e.g. if no further negative reduced cost paths can be identified. The latter ensures that optimality of the CRG procedure is not affected.

Although the definition of the penalties was motivated by practical considerations, the described problem can also be seen in the light of dual variable stabilization theory (see e.g. Merle et al., 1999; Ben Amor et al., 2009); the initial zero dual variable values for dual variables λ_a and ϕ_{ak} are likely to be very far from their optimal values and it may take a very large number of iterations until they stabilize. The dual penalties $\tilde{\lambda}_a$ aim at providing a much better initial estimate of dual variable values λ_a.

2.5.5 Obtaining feasible integer solutions

The final goal of each improvement iteration of method LSNDSP-CGM is to obtain an improved scheduled liner shipping network. The generation of feasible integer solutions based on the latest RMP is divided into two steps.

First, we use a standard MIP solver to generate feasible networks defined by integer y_a^e variable values. We impose a time limit that is defined as a factor F_{MIP} times the accumulated time used to solve the RMP during the preceding CRG phase (lines 3 and 8 of Algorithm 2). We conjecture that the accumulated time for solving the RMP during the column and row generation phase is a good indicator for the difficulty of model M. An advantage of a dynamic time limit is that it adapts to different model sizes in general as well as during the execution of method LSNDSP-CGM; the model size may vary significantly between iterations depending on the outcome of the network fixing procedure (Section 2.5.1). The integer solutions found during the MIP solving phase are stored in a solution pool X.

Second, we apply the MIP polishing heuristic by Rothberg (2007), aiming at diversifying and enlarging the set of feasible scheduled networks X. Based on the previously generated MIP solutions and the corresponding MIP search tree, the MIP polishing heuristic mutates and combines existing feasible solutions, fixes common variable values and explores a fixed number of nodes in the resulting sub-MIP tree. Even though the method is not problem specific, it implicitly considers the problem structure due to its tight interaction with the MIP search tree. The MIP polishing heuristic is a randomized algorithm. Equivalent to the MIP solving phase we impose a dynamic time limit F_{Pol} as a factor of the accumulated time used to solve the RMP during the preceding CRG phase.

2.5.6 Cargo allocation for feasible networks

The scheduled networks generated during the MIP+polishing phase are based on the restricted set Ω' of cargo paths. For any solution $\chi \in X$ there may exist further negative reduced cost cargo paths $j \in \Omega \setminus \Omega'$ with the potential to improve a solution. In order to identify them, we solve a CAP for each of the solutions in the pool X.

For each solution $\chi \in X$ we fix the binary variables y_a^e that correspond to the services $s \in S_\chi$ to one (and all other variables to zero) and evaluate whether further negative reduced cost cargo paths exist. The resulting problem is a time constrained multi-cargo flow (MCF) problem that can be solved efficiently using column generation as described in Section 2.5.2.

The solution χ' with the lowest objective function value among all solutions $\chi \in X$ defines the best solution of the current improvement iteration.

2.5.7 Tabu constraints

During the MIP+polishing phase the same networks may be generated in subsequent iterations. In order to avoid it and improve diversification, we add a tabu constraint to model M at the end of each improvement iteration. The constraint forbids the combination of redesigned services (excluding the fixed services) of the best solution $\chi' \in X$ in subsequent improvement iterations. The constraints are of the form $\sum_{e \in E \setminus E'} \sum_{a \in \hat{A}_e} y_a^e \leq | \cup_{e \in E \setminus E'} \hat{A}_e| - 1$ with $\hat{A}_e$ denoting the arcs that are used by any service of vessel class e. Thus, the constraints forbid only the set of services that has been re-designed during an iteration. Tabu constraints in set $\mathcal{T}$ are activated and deactivated dynamically at each iteration depending on which part of the network is fixed. Otherwise a tabu constraint might be in conflict with the network fixing constraints. Whenever a new global best solution is found, the set $\mathcal{T}$ of tabu constraints is deleted.

2.6 Computational results

We implemented the solution algorithm in C++ and used the Lemon graph library (Dezsõ et al., 2011) to model the underlying graphs. CPLEX 12.7 was used to solve the LP relaxations as well as to generate MIP solutions. All computational experiments were run on a system with a Xeon E5-2680 2.8GHz processor and 48 GB memory.

In Section 2.6.1 we provide a summary of the data instances and their corresponding LSNDSP graph and model instances. We present and discuss results of the LSNDSP-CGM for different instances in Section 2.6.2. In Section 2.6.3 we analyse the implications of integrating scheduling into liner shipping network design and compare our results with those obtained under the simplifying assumption of constant transshipment times; for the latter case we present both our own experiments as well as results from a state-of-the-art algorithm for unscheduled network design from the literature. Finally, Section 2.6.4 provides insights about the algorithm performance for different instances.

2.6.1 Data instances

The LSNDSP-CGM was tested on data instances from the publicly available LINER-LIB benchmark suite (`http://www.linerlib.org`). Table 2.1 provides a short summary of the instances; for a detailed description we refer the reader to Brouer et al. (2014a). In order to fairly compare the performance of LSNDSP-CGM with results from the literature, we have used a demand rejection penalty of 1000 USD, a bunker price of 600 USD per ton and a constant port stay time of 24 hours for all ports as consistently used in previous studies (see e.g. Karsten et al., 2017b). A minimum transshipment time of 48 hours between arrival of the unloading and departure of the loading vessel was assumed. The maximum transshipment time is set to the minimum transshipment time plus one week, given the weekly service frequency.

A time discretization of 12 hours was used for the graph model, representing a good trade-off between solution quality and problem tractability. A finer time discretization implies much larger graphs, whereas a coarser time discretization can be too restrictive and result in solutions of poor quality, as preliminary tests have shown. The number of arcs and vertices of the resulting time-space graphs and the theoretical size of model (2.1)–(2.8) corresponding to each data instance are reported in Table 2.2. It is noteworthy that even the smallest instance of the LINER-LIB translates into a large service network design instance in terms of vertices and arcs. In the case of instance `WorldSmall` the large size of the resulting LSNDSP instance caused the LSNDSP-CGM to run out of memory. In the remainder of this section we therefore report results only for the instances `Baltic`, `WAF`, `Mediterranean` and `Pacific`.

2.6.2 LSNDSP-CGM results

In a first test setting we ran method LSNDSP-CGM in order to construct liner shipping networks from scratch. The method was run with the same parameter setting on all instances. The number of CRG iterations per improvement iteration was set to $I_{\text{crg}}^{\max} = 3$ and the maximum number of cargo paths generated per demand and CRG iteration was set to $N_{\text{crg}}^{\max} = 5$. The time limit for the MIP solve call during each improvement iteration was set to be equal to the accumulated time spent on solving all RMPs during the preceeding CRG phase ($F_{\text{MIP}} = 1.0$) and the MIP polishing heuristic was given three times the accumulated time spent on solving all RMPs ($F_{\text{Pol}} = 3.0$). The maximum number of solutions to be stored in the solution pool during the MIP solving and polishing phase was set to 50. No time limit was imposed on the solution of the CAP, i.e. the CAP was solved to optimality for each solution in the solution pool

Instance	Description	Ports	Legs	Vessel Classes	Vessels	Demands
Baltic	Baltic sea, single hub Bremerhaven	12	132	2	6	22
WAF	West Africa, single hub Algeciras	20	402	2	42	37
Mediterranean	Mediterranean, multi-hub	39	1,482	3	20	365
Pacific	Trade lane Asia and US west coast	45	2,122	4	100	722
WorldSmall	World, 47 main ports	47	3,142	6	263	1764

Table 2.1: Data instances of the LINER-LIB

Instance	Graph $G(V, A)$				Model	
	$\lvert V\rvert$	$\lvert A\rvert$	$\lvert A^S\rvert$	$\lvert A^T\rvert$	constraints	binary vars
Baltic	168	7,812	5,460	2,352	5,988	8,652
WAF	280	42,168	38,248	3,920	39,127	51,016
Mediterranean	546	108,206	101,108	7,644	103,660	165,816
Pacific	630	542,262	534,072	8,820	537,948	1,044,708
WorldSmall	658	1,516,844	1,507,632	9,212	1,514,008	4,130,112

Table 2.2: Graph and model properties for different data instances, based on a time discretization of 12 hours.

Instance	Obj. val.	Deployed cap. (%)	Transp. vol. (%)	Trans-shipped vol. (%)	Avg. cap. util. (%)	Avg. Speed (kn)	Butterfly ser-vices (%)	Complex ser-vices (%)	Time limit (sec.)
Baltic									
Best	$-2.84{\cdot}10^5$	100.0	92.1	0.0	73.4	11.9	50.0	50.0	900
Average	$-2.24{\cdot}10^5$	100.0	92.0	0.9	73.7	12.3	48.1	37.0	900
WAF									
Best	$-5.92{\cdot}10^6$	94.4	96.8	17.7	57.2	10.8	20.0	40.0	3600
Average	$-5.76{\cdot}10^6$	95.5	96.3	23.7	57.3	10.8	22.9	37.1	3600
Mediterranean									
Best	$2.54{\cdot}10^6$	100.0	77.2	49.6	55.2	11.1	0.0	42.9	14400
Average	$2.73{\cdot}10^6$	96.2	73.8	43.3	53.7	10.9	25.0	33.8	14400
Pacific									
Best	$2.69{\cdot}10^6$	93.7	85.4	20.2	75.4	13.7	12.5	25.0	28800
Average	$3.71{\cdot}10^6$	98.2	84.9	23.8	72.9	13.4	14.1	29.7	28800

Table 2.3: Key statistics for the LINER-LIB instances solved by the LSNDSP-CGM method. Best and average results are reported for each instance. Reported values are: objective value (in USD), deployed capacity as percentage of total available TEU capacity, transported volume as percentage of total number of containers, transshipped volume as percentage of transported volume, average vessel utilization (weighted by sailing distance), average speed in knots (weighted by sailing distance), number of butterfly services as percentage of total number of services, number of complex services as percentage of total number of services and run time limit

at each improvement iteration. We used a constant dual penalty multiplier of $\eta = 2.0$ in all experiments. Different time limits $T^{\max}$ on the total runtime of LSNDSP-CGM were imposed for different instances according to their size. The time limit for each instance is reported in the last column of Table 2.3.

As method LSNDSP-CGM is a randomized algorithm, we ran it 12 times for each data instance. Table 2.3 reports key statistics for the best run and the average of all runs for each instance. A first conclusion that we can draw from the reported key statistics is that the instances and the resulting scheduled networks are quite heterogeneous. Whereas basically no cargo is transshipped in the liner shipping networks found for instance `Baltic`, around half of all transported containers are transshipped at least once in the solutions for `Mediterranean`. The average capacity utilization rates vary between 53.7% and 73.7% and average sailing speeds range from 10.8 knots to 13.4 knots.

The third-to-last and the second-to-last column of Table 2.3 display the share of butterfly and complex services among all services of a solution. The difference to 100% represents the share of simple services. The high share of complex liner services in the final solutions suggests that these bear large potential to improve liner shipping networks. The capability of modeling complex liner services straightforwardly is one of the distinguishing feature of our proposed model for the LSNDSP.

Instance	LSNDSP-CGM		Karsten et al. (2017b)
	exact transship. time	48h transship. time	48h transship. time
Baltic			
Best	$-2.84 \cdot 10^5$	$-2.84 \cdot 10^5$	$-0.05 \cdot 10^5$
Average	$-2.24 \cdot 10^5$	$-2.08 \cdot 10^5$	$1.74 \cdot 10^5$
WAF			
Best	$-5.92 \cdot 10^6$	$-5.90 \cdot 10^6$	$-5.48 \cdot 10^6$
Average	$-5.76 \cdot 10^6$	$-5.77 \cdot 10^6$	$-4.89 \cdot 10^6$
Mediterranean			
Best	$2.54 \cdot 10^6$	$2.11 \cdot 10^6$	$2.19 \cdot 10^6$
Average	$2.73 \cdot 10^6$	$2.33 \cdot 10^6$	$2.65 \cdot 10^6$
Pacific			
Best	$2.69 \cdot 10^6$	$-0.33 \cdot 10^6$	$1.13 \cdot 10^6$
Average	$3.71 \cdot 10^6$	$1.06 \cdot 10^6$	$3.44 \cdot 10^6$

Table 2.4: Best and average objective function values (in USD) per instance, comparing results obtained by algorithm LSNDSP-CGM under exact and approximated (48h) transshipment times with results from Karsten et al. (2017b)

2.6.3 Scheduled vs. unscheduled network design

To investigate the implications of integrating scheduling into the liner shipping network design problem, we have repeated the first set of computational experiments described in Section 2.6.2 under the simplifying assumption of a constant transshipment time of 48 hours for any transshipment. This was implemented by setting the duration of all transshipment arcs $a \in A^T$ to 48 hours, independent of the weekly time associated with their origin and destination vertices. As discussed in the introduction, this simplified setting resembles the large majority of problem formulations for the liner shipping network design problem in the literature. Furthermore, testing our solution method LSNDSP-CGM under these assumptions allows us to directly compare our method against algorithms proposed in the literature.

Table 2.4 displays the objective function values obtained by LSNDSP-CGM for the original LSNDSP as well as for the LSNDSP under the simplifying assumption of constant transshipment times. In the third column, the objective function values found by the current state-of-the-art solution method for the liner shipping network design problem by Karsten et al. (2017b) are reported. We make two key observations.

First, the average objective function values obtained by method LSNDSP-CGM differ significantly between scheduled and unscheduled network design (p-value < 0.0001) for the two large instances. Much better (i.e. lower) objective function values are obtained in case transshipment times are approximated by a constant of 48 hours. This is not surprising, because the *optimal* solution obtained under the assumption of constant transshipment times of 48 hours in fact represents a lower bound for the optimal solution of the LSNDSP. For the two smaller instances the differences in the objective function values are not significant. We will discuss the implications

Instance	Deployed cap. (%)		Transported volume (%)		Transported volume (FEU)				Capacity util. (%)		Speed (kn)	
					direct		transshipped					
	exact	approx	exact	approx	exact	approx	exact	approx	exact	approx	exact	approx
Baltic	100.0	100.0	92.0	92.4	4477	4524	37	5	73.7	72.8	12.3	12.2
WAF	95.5	94.2	96.3	95.5	5406	5296	2739	2874	57.3	57.4	10.8	10.8
Mediterranean	96.2	96.8	73.8	79.3	3523	2667	2002	3400	53.7	60.6	10.9	11.0
Pacific	98.2	97.9	84.9	89.2	29080	24362	8419	14375	72.9	74.8	13.4	13.6

Table 2.5: Comparison of key statistics of scheduled (exact) vs. unscheduled (approx.) network design. Reported values represent averages over 12 runs. For underlined pairs of values the difference is statistically significant at the 0.01 level (based on Welch's t-test).

of neglecting scheduling in more detail shortly.

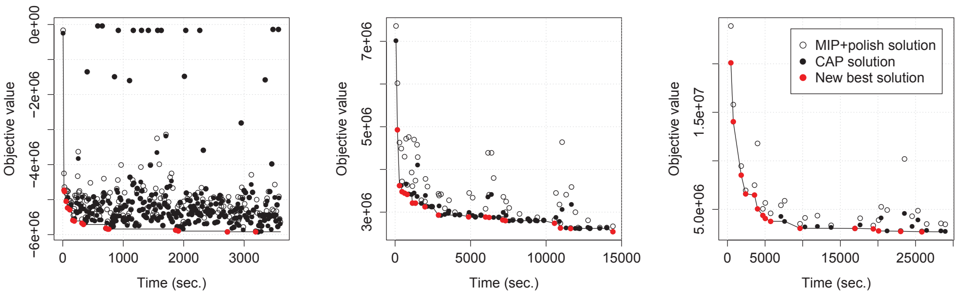
Second, we observe that for the unscheduled network design problem, method LSNDSP-CGM consistently finds better solutions than the state-of-the-art algorithm from the literature for all addressed instances. It is, however, fair to note that our algorithm was given more time per instance (e.g. 28800 seconds vs. 3600 seconds for `Pacific` in Karsten et al., 2017b).

Table 2.5 compares different solution statistics for the LSNDSP against the solutions obtained under the simplifying assumption of constant transshipment times. For the two smaller instances, none of the reported statistics differ between the LSNDSP and the LSNDSP under the assumption of constant transshipment times. For the `Baltic` instance, transshipments almost do not occur. For the `WAF` instance the transit time limits of demands appear to be loose enough to not matter, as almost all (97%) of the rejected demand is rejected because the origin or destination port are not served by any service and not because of transit time restrictions.

The picture looks quite different for the larger instances. A significantly larger amount of containers is transported and a significantly higher vessel capacity utilization rate is achieved under the simplifying constant transshipment time assumption. While the increased amount of transshipped containers (+70%) under the constant transshipment time assumption is in line with what one would expect, we also observe that the amount of directly transported containers is *lower* under the constant transshipment time assumption. It appears that the artificial fixing of transshipment time to 48 hours actually allows containers to use routes that include a transshipment, while these routes would take too long otherwise. Indeed, the higher utilization rates may partly result from an increased absolute amount of transported containers and partly from a higher level of consolidation, because more transportation time is available per demand due to underestimated transshipment times. The results reflect the common trade-off: a higher level of consolidation implies less direct routes and therefore higher cargo transit times and vice versa.

Instance	Time limit	Iterations	Time per iter.	Running time (%) Cmp pricing	RMP	MIP+ polishing	CAP	Solutions per iter.
Baltic	900	274	3.3	1.0	17.6	71.0	8.8	2.7
WAF	3600	329	11.0	0.9	14.2	58.0	25.7	10.9
Mediterranean	14400	80	179.1	5.6	17.5	70.0	6.6	12.9
Pacific	28800	37	785.5	10.2	14.1	56.8	18.7	23.6

Table 2.6: Algorithm run statistics. Values represent averages of 12 runs per instance. Time limit (sec.), number of improvement iterations, avg. time per iteration (sec.), time for cargo path pricing (%), time for solving RMPs (%), time for generating integer solutions (%), time for solving Cargo Allocation Problems (%) and avg. number of solutions found per iteration. The difference of the sum of running times (%) to 100% attributes to the other algorithm components.



(a) WAF (b) Mediterranean (c) Pacific

Figure 2.4: Algorithm convergence over time for the best runs of WAF, Mediterranean and Pacific. The displayed MIP and CAP objective function values correspond to the best MIP and CAP objective function value among all solutions in the solution pool of an improvement iteration.

2.6.4 Analysis of solution algorithm performance

The method LSNDSP-CGM proves to work consistently well on different types and sizes of liner shipping network design problem instances. Nevertheless, the instance size affects the number of iterations that can be performed during a given time limit as well as the absolute time spent on the different algorithm phases, as the algorithm statistics in Table 2.6 show. On average algorithm LSNDSP-CGM spends the largest amount of time during the MIP+polishing phase, which (by definition of the dynamic time limits F_{MIP} and F_{Pol}) is four times as large as the time spent on solving RMPs. Despite the large number of cargo path pricing problems solved at each CRG iteration, the accumulated solution time is almost negligible for the small instances and accounts for only 10% of the total runtime even in case of instance `Pacific`.

Figures 2.4a to 2.4c visualize the algorithm convergence over time for the best run of each instance. For smaller instances, the search space relative to the total solution space during an

Instance	Iters (%) w. improvement	Avg. improvement	Avg. improvement (%)
Baltic	49.0	51768	11.8
WAF	75.3	129897	2.5
Mediterranean	85.9	403203	9.4
Pacific	99.3	1917247	17.5

Table 2.7: Impact of solving a CAP for each solution found during the MIP+polishing phase. The first column reports the relative number of iterations in which the solution of the CAP was strictly better than the solution of the MIP+polishing phase. The second and third column report the absolute and relative average improvement, including the iterations without any improvement (averaged over all iterations of all runs).

improvement iteration is larger due to the smaller number of vessel classes and services. This allows the objective function value of networks found at the end of an improvement iteration to deviate much more from the current best network and explains the larger fluctuation in objective function values for the small instance `WAF` compared to the large instances `Mediterranean` and `Pacific`.

Figure 2.4 also reveals the importance of solving the CAP for each of the networks found during the MIP+polishing phase; in the large majority of cases and particularly for larger instances, the step identifies additional profitable cargo paths that lead to a significant improvement of the best objective function value of the MIP+polishing phase. Table 2.7 reports the relative number of iterations with an improvement and the average improvement per iteration of the best CAP solution compared to the best MIP+polishing solution of the same iteration. The reported values are calculated over all iterations of all runs. Like indicated by Figure 2.4, the approach pays off the most for larger instances.

2.7 Concluding remarks and future research

We proposed a graph and model formulation for the integrated liner shipping network design and scheduling problem. By integrating scheduling into the classic liner shipping network design problem we account for the fact that in practice network key performance indicators like cargo transit times depend substantially on the synchronization of liner services. Additionally, the proposed model is very rich in that it allows to incorporate details which previously have only been addressed in isolated cases.

We have analyzed the relevance of integrating scheduling into the classic liner shipping network design problem by testing our model against the prevalent assumption of constant transshipment times. The results show that if the service level matters, the assumption may underestimate both transshipment times and consequently transit times of cargo. Networks or parts of networks that are designed based on the simplifying assumption may finally not be able to meet shippers service level requirements. Furthermore, key figures like average vessel utilization may turn out to be significantly overestimated, as the results indicate.

The richness of the proposed model comes at the cost of an increased problem complexity. We proposed an iterative solution algorithm that combines heuristic and advanced linear programming techniques into a powerful matheuristic for constructing or improving scheduled liner shipping networks. We demonstrated the effectiveness of the approach by benchmarking it on the publicly available liner shipping network design data instances. Under equal assumptions LSNDSP-CGM consistently finds better solutions for all addressed instances.

As a next step we plan to enrich the model to further close the gap to operational practice. One practically relevant extension is the consideration of vessel payloads for fuel consumption and speed optimization, as load factors and related fuel consumption may differ significantly between headhaul and backhaul trades. Similarly, cargo carriers can only fully exploit the potential savings of an optimized schedule if it is robust against interruptions; delays in liner shipping are not an exception and often lie outside the cargo carriers' sphere of influence. Quantifying and including schedule reliability into the decision making process is thus a worthy and practically relevant research direction.

The further development of the proposed solution method represents another research avenue. We believe that different definitions of the local search space may allow to improve efficiency and scalability of the algorithm. Given the very positive performance of the proposed LSNDSP-CGM on comparatively large instances, it may be worth examining if the method can be generalized and adapted towards other variations of service network design problems.

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Chapter 3

Integrated Liner Shipping Network Design and Scheduling: Addendum

3.1 Introduction

The proposed model and solution algorithm for the LSNDSP presented in Chapter 2 represent the distillate of an extensive evaluation and testing of different models, algorithm frameworks and solution approaches. We believe that one of the main advantages of our presented modeling approach for the LSNDSP is its ability to accommodate additional restrictions and features without destroying its structure and thus without the need for fundamentally different solution approaches.

In Section 3.2 we back up this hypothesis by showing how selected model extensions can be integrated into the LSNDSP model without invalidating the solution method. The extensions include the integration of empty container repositioning and the consideration of more complex revenue functions of transit time. In Section 3.3 we briefly describe a service based model formulation that allows for complex restrictions imposed on the service design.

3.2 Model extensions

A nice property of the model and solution method for the liner shipping network design and scheduling problem presented in Chapter 2 are their adaptability and customizability. Below we present different extensions and describe how they can be implemented without destroying the structure of the original model or invalidating the solution method.

In Section 3.2.1 we present an extension that allows to explicitly consider the repositioning of empty containers in the network design problem. The required modification of both the master and the pricing problems are described in detail. The explicit consideration of refrigerated containers and corresponding capacity constraints is addressed in Section 3.2.2. In Section 3.2.3 we show how any non-increasing, non-negative and possibly non-continuous revenue function of transit time can be handled by the model and solution method. In Section 3.2.4 we present a strategy to possibly reduce the number of one-to-all cargo path pricing problems and show that significant reductions are possible for the two smallest LINER-LIB data instances.

3.2.1 Empty container repositioning

The model and solution method presented for the LSNDSP in Chapter 2 is driven by the demand for transportation of *laden* containers. The global import and export trade is, however, highly unbalanced, especially on the Asia-North America and Asia-Europe trade lines. In 2005, the number of laden containers (in TEUs) shipped from Asia to the US was more than three times larger than the trade volume in the opposite direction (Song and Carter, 2009). As a consequence, empty containers have to be repositioned from ports and regions with low export and high import volumes to ports and regions where export volumes exceed import volumes. The share of empty containers is anything but negligible; in 2006, empty containers represented around 20% of all shipped containers (Drewry Shipping Consultants, 2007). Alderton (2011) presents an even higher estimate of 30% and states that on some trades the share of empty containers can exceed 80%. Empty containers represent a significant cost factor for liner shipping companies, as their transport does not generate revenues.

Consequently, the problem has received increasing attention during the past decade. Brouer et al. (2011) propose a path-flow formulation for the liner shipping cargo allocation problem with empty container repositioning. The goal is to decide both laden and empty container flows in a given liner shipping network such that container in- and out-flow are balanced at each port. Shintani et al. (2007) present a model and genetic algorithm to design a single shipping route while explicitly considering empty container repositioning. Meng and Wang (2011) address a network design problem that considers both laden and empty container flows. Sailing routes are not designed, however, but selected from a pool of at most 60 liner services.

Song and Carter (2009) present and evaluate two different strategies – *internal coordination* and *external container sharing* – to address the problem of balancing container flows in a liner shipping network. Internal coordination refers to the balancing of container flows among different routes by a single carrier. External container sharing allows the exchange of empty containers among different carriers, possibly at some cost. Both strategies can be applied individually or combined. Not surprisingly, the authors find the combined strategy to perform best; among the two, however, internal coordination is found to have the largest impact on savings.

We present an extension to the LSNDSP that allows for both *internal coordination* as well as *external container sharing*. We demonstrate how empty container flows to balance container import and export at each port can be integrated into the LSNDSP model (2.1)–(2.8) (see Chapter 2). More importantly, we also show how the proposed method LSNDSP-CGM can be extended to handle the increased complexity straightforwardly and without losing the method's effectiveness.

We first define a set $\tilde{K}$ of demands of empty containers between all ordered pairs of ports and let Ω^k denote the set of all empty container paths for a given demand k. We use $o(k)$ and $d(k)$ to denote the origin and destination port of a demand k. In fact, empty container paths are mathematically equivalent to laden cargo paths; empty container paths do, however, not generate any revenue, no transit time restrictions apply and, needless to say, no penalty for not transporting empty containers exists.

In order ensure an equal number of incoming and outgoing containers at each port (within a week), we add flow balancing constraints to model (2.1)–(2.8):

$$\sum_{k \in K \cup \tilde{K}} \sum_{\{j \in \Omega^k : o(k)=p\}} x_j^k - \sum_{k \in K \cup \tilde{K}} \sum_{\{j \in \Omega^k : d(k)=p\}} x_j^k + \hat{x}_p^- - \hat{x}_p^+ = 0, \qquad \forall p \in P \qquad [\mu] \tag{3.1}$$

Constraints (3.1) are defined over the set of ports and require that the weekly import of containers equals the weekly export of containers. Imbalances for laden containers can be compensated in two ways: by repositioning empty containers internally, represented by the path variables $\tilde{x}_j^k$; and by leasing containers from or to an external supplier or consumer, represented by non-negative continuous variables $\hat{x}_p^+$ and $\hat{x}_p^-$ defined over the set of ports. Non-zero values for $\hat{x}_p^+$ or $\hat{x}_p^-$ imply a lease *from* an external supplier or a lease *to* an external demander at port $p \in P$, respectively.

The original objective function (2.1) is extended by adding the cost for empty container paths and the cost or revenue (if any) for leasing containers from or to external suppliers or consumers, respectively:

$$\min \quad \sum_{a \in A^S} \sum_{e \in E_a} c_a^e y_a^e + \sum_{k \in K \cup \tilde{K}} \sum_{j \in \Omega^k} (c_j^k - p^k) x_j^k + \sum_{k \in K} q^k p^k + \sum_{p \in P} \hat{c}_p^+ \hat{x}_p^+ + \sum_{p \in P} \hat{c}_p^- \hat{x}_p^- \tag{3.2}$$

Adding constraints (3.1) to model (2.1)–(2.8) also affects the reduced cost of cargo paths through dual variable values μ_p. The reduced cost of a cargo path $j \in \Omega^k, k \in K \cup \tilde{K}$ is re-defined as $\bar{c}_j^k := -z_k - \alpha_k - \mu_{o(k)} + \mu_{d(k)} + \sum_{a \in \bar{A}^k} b_{ja} \bar{c}_a$. The label setting algorithm described in detail in Section 2.5.3 remains unaffected, except that the upper bounds on the cost component of a label have to be updated to $\bar{C}_{kv} := z_k + \alpha_k + \mu_{o(k)} - \mu_{d(k)}$.

Although the number of pricing problems to solve at each iteration theoretically grows by $|\tilde{K}|$ if empty container repositioning is considered, the number is likely to be much smaller in practice. As all arcs have a non-negative reduced cost, the pricing problem of empty container demand $k \in K \cup \tilde{K}$ needs to be solved only in case $z_k + \alpha_k + \mu_{o(k)} - \mu_{d(k)} > 0$; as $z_k = 0$ and $\alpha_k = 0$ for all empty container demands $k \in \tilde{K}$, the question whether to solve the pricing problem of a particular empty container demand $k \in \tilde{K}$ thus solely depends on the dual variable values $\mu_{o(k)}$ and $\mu_{d(k)}$.

To conclude, the Liner Shipping Network Design and Scheduling Problem can be extended to include empty container repositioning without increasing the problem complexity.

3.2.2 Explicit consideration of refrigerated containers

The LINER-LIB benchmark data instances by Brouer et al. (2014) do not distinguish between different types of containers. While it is a reasonable assumption in general, there are some practical implications related to refrigerated containers. Refrigerated containers, also called *reefers*, usually rely on external power while being on a vessel. Not all container slots are provided with power points and hence the capacity for reefers is different from the capacity for non-refrigerated containers.

If origin-destination demands for reefers were known, the following extension allows to handle capacities for refrigerated (or any other type of) containers explicitly. Let us assume that the set of demands can be split into demands of non-refrigerated and refrigerated containers $K = K^N \cup K^R$. Mixed origin-destination demands can simply be split into two demands of only non-reefers and only reefers. Let U_R^e denote the reefer capacity of a vessel of vessel class $e \in E$.

An additional set of arc capacity constraints ensures that reefer capacities are not violated:

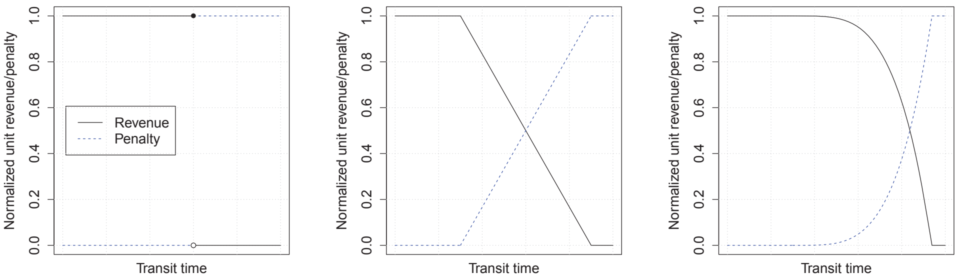
$$\sum_{k \in K^R} \sum_{j \in \Omega^k} b_{ja} x_j^k - \sum_{e \in E_a} U_R^e y_a^e \leq 0, \qquad \forall a \in A^S \qquad [\lambda^R] \tag{3.3}$$

The $\lambda^{\mathbf{R}}$ dual variable vector associated with constraints (3.3) is equivalent to the λ dual variable vector associated with the original arc capacity constraints. Their consideration in, for example, the reduced cost of cargo path pricing problems is straightforward.

3.2.3 Transit time dependent revenue functions

The results presented for the LSNDSP in Chapter 2 are based on LINER-LIB data. One of the implicit assumptions of the LINER-LIB data instances is that hard transit time limits exist for demands and that the revenue is independent of the transit time otherwise; that is, the shipping company only earns money by shipping a particular demand, if the transit time is satisfied. If it is exceeded by an hour, no revenue is earned.

Wang et al. (2013) conduct experiments based on the hypothesis that revenue is a decreasing function of transit time, however, no support for that hypothesis is provided; Psaraftis (2017) criticizes their work as an example for his *cart before the horse* conjecture, stating that published research more and more often is driven by the methodology rather than by the addressed problem. Unfortunately, we are not aware of any work that studies revenue functions for container transport or evaluates common assumptions against empirical data. Anyhow, below we show how any non-increasing (possibly non-continuous) revenue function of transit time can be handled by the model and solution method proposed in Chapter 2.



(a) Constant revenue function with transit time limit

(b) Piecewise linear revenue function

(c) Non-linear revenue function

Figure 3.1: Examples of non-increasing transit time dependent revenue functions

Let $r_k(t)$ denote a non-increasing, non-negative and possibly non-continuous revenue function of demand $k \in K$ that depends on transit time t. Without loss of generality, we can normalize $r_k(t)$ such that $r_k(t) \in [0, 1]$. Let $p_k(t) := 1 - r_k(t)$ denote a corresponding penalty function; it represents the value by which the nominal (maximum) revenue of a demand is penalized for a given transit time. Thus, function $p_k(t)$ is non-decreasing. Figure 3.1 illustrates different revenue functions and their corresponding penalty functions.

Any hard transit time limit $t_k^{\max}$ of a demand $k \in K$ can be modeled in the revenue function $r_k(t)$ by defining the function such that $r_k(t) := 0$ if $t > t^{\max}$. Figure 3.1a shows a constant revenue function with a hard transit time limit. The function is non-continuous due to the transit time limit. Equivalently, the transit time limit can be interpreted as the smallest transit time at which the revenue function $r_k(t)$ takes value 0, i.e. $t^{\max} = \arg\min_t t \in [0; \infty] : r_k(t) = 0$.

If the revenue function is independent of the transit time, we can use a standard resource extension function of the form $f_{uv}(C_u) = C_u + c_{uv}$, as done in Chapter 2 (we omit index k for readability). C_u denotes the value of the resource (here: cost) at vertex u. In order to handle more complex revenue functions as described above, we extend the standard resource extension function by a term representing the lost revenue due to transit time:

$$f_{uv}(C_u, T_u) = C_u + c_{uv} + p(T_u + t_{uv}) - p(T_u) \tag{3.4}$$

The additional term $p(T_u + t_{uv}) - p(T_u)$ represents the loss in revenue (penalty) due to extending the transit time from T_u to $T_u + t_{uv}$ and also depends on the transit time associated with the label at vertex u that is to be extended. As the time $t_{uv} \geq 0$ and as $p(t)$ is non-decreasing, the term $p(T_u + t_{uv}) - p(T_u)$ is always non-negative.

The complexity of the resource constrained shortest path problem remains unchanged and the problem can be solved efficiently in case revenue is a non-increasing function of transit time.

3.2.4 Reducing the number of cargo path pricing subproblems to solve

In case no valid inequalities (2.9) are added or, more precisely, if all dual values $\phi_{ak} = 0$, we can solve a one-to-all RCSPP for each origin port $p \in P$ instead of solving one commodity path pricing subproblem per commodity, as proposed by Karsten et al. (2015).

We can further reduce the number of commodity path pricing subproblems based on the following observation: Finding the shortest path (with resource constraints) from an origin to a destination on a graph G is equivalent to finding the shortest path (with resource constraints) from the destination to the origin on reversed graph $R(G)$, where all (directed) arcs are reversed. The number of subproblems depends on the actual set of demands and may differ, depending on whether we solve origin-to-all RCSPPs on the original graph or destination-to-all RCSPPs on the reversed graph. By mixing both types of subproblems such that every demand is covered by at least one subproblem we can obtain a minimal number of subproblems to solve.

The problem of finding the minimal number of subproblems to solve can be formulated as a binary integer program. Let w_p^o denote binary variables that indicate if a one-to-all RCSPP problem is solved for port p on graph G ($w_p^o = 1$) or not ($w_p^o = 0$). Equivalently, let w_p^d denote binary variables that indicate if a one-to-all RCSPP problem is solved for port p on *reversed* graph $R(G)$ ($w_p^d = 1$) or not ($w_p^d = 0$). We use notation $o(k)$ and $d(k)$ to denote the origin and destination port, respectively, of a demand $k \in K$. The problem of minimizing the number of subproblems can be formulated as

$$\min \quad \sum_{p \in P} (w_p^o + w_p^d) \tag{3.5}$$

$$\text{s.t.} \quad w_{o(k)}^o + w_{d(k)}^d \geq 1, \qquad \forall k \in K \tag{3.6}$$

$$w_p^o \in \{0,1\}, \quad \forall p \in P \tag{3.7}$$

$$w_p^d \in \{0,1\}, \quad \forall p \in P \tag{3.8}$$

The objective function (3.5) minimizes the total number of subproblems to be solved, consisting of the number of subproblems defined over graph G and the number of subproblems defined over reversed graph $R(G)$. Constraints (3.6) require that each demand is covered by at least one subproblem. It requires each demand to be either part of an origin-to-all RCSPP (with the demand's origin port equal to the RCSPP's origin port) or to be part of an destination-to-all RCSPP on the reversed graph $R(G)$ (with the demand's destination port equal to the RCSPP's destination port). Constraints (3.7) and (3.8) require the decision variables to be binary.

Table 3.1 shows the resulting minimal number of commodity path pricing subproblems (RC-SPPs) to be solved for each data instance from the LINER-LIB. The two smallest instances `Baltic` and `WAF` are characterized as *hub instances*, i.e. all demands either originate or end at a single hub port (Bremerhaven and Algeciras, respectively). Thus, by solving only two subproblems, the negative reduced cost paths of all demand's can be obtained in case of the smallest two

Instance	# ports	# demands	# RCSPPs	# RCSPPs (min)
Baltic	12	22	12	2
WAF	20	37	19	2
Mediterranean	39	365	36	35
Pacific	45	722	45	44
EuropeAsia	111	4000	111	111
WorldSmall	47	1764	47	47
WorldLarge	201	9622	197	197

Table 3.1: Number of RCSPPs to solve. RCSPPs (min) denotes the minimized number of subproblems that have to be solved.

instances. The demand patterns of the larger instances are more realistic and cover arbitrary origin-destination pairs. For these instances, however, it turns out that the potential of reducing the number of subproblems to solve is almost zero.

3.3 Service based model reformulation

We have not imposed any restriction on the design of liner services in the LSNDSP model presented in Chapter 2, except for the maximum number of available vessels per vessel class. If the presented model and solution approach are used as a decision support tool in a practical setting, the decision maker *may* want to design new service(s) under additional restrictions. These could, for example, restrict the duration of a service or, equivalently, the number of vessels deployed on a service; the decision maker may require some ports to be excluded from or covered by a particular service, possibly in a given order; or the decision maker may want to limit the number of port calls between two selected ports, and so on.

The modeling and integration of any set of general service design restrictions into model (2.1)–(2.8) is not straightforward. If feasible at all, it most likely requires additional constraints or variables and destroys the structure of the problem, making it much harder to solve. A simple and common way of handling additional restrictions on the service design is to switch from an *arc*-based formulation (with respect to the service design variables y_a) to a *service*-based model formulation. That is, the design of liner services is done by solving pricing subproblems and the master model solely selects the services to be part in a solution.

Let S denote the set of all feasible liner services and let y_s^e be a binary decision variable denoting whether a particular service $s \in S$ is operated by vessel class $e \in E$ ($y_s^e = 1$) or not ($y_s^e = 0$). Binary parameters b_{sv} and b_{sa} denote whether a service $s \in S$ does or does not use a particular vertex $v \in V$ or arc $a \in A^S$, respectively.

$$\min \quad \sum_{e\in E}\sum_{s\in S^e} c_s^e y_s^e + \sum_{k\in K}\sum_{j\in\Omega^k} c_j^k x_j^k + \sum_{k\in K}\left(q^k - \sum_{j\in\Omega^k} x_j^k\right)p^k \tag{3.9}$$

$$\text{s.t.} \quad \sum_{j\in\Omega^k} x_j^k \le q^k, \quad \forall k\in K \quad [\alpha] \tag{3.10}$$

$$\sum_{e\in E_v}\sum_{s\in S^e} b_{sv} y_s^e \le 1, \quad \forall v\in V \quad [\beta] \tag{3.11}$$

$$\sum_{k\in K_a}\sum_{j\in\Omega^k} b_{ja} x_j^k - \sum_{e\in E_a}\sum_{s\in S^e} b_{sa} U^e y_s^e \le 0, \quad \forall a\in A^S \quad [\lambda] \tag{3.12}$$

$$\sum_{s\in S^e} n_s^e y_s^e \le N^e, \quad \forall e\in E \quad [\pi] \tag{3.13}$$

$$x_j^k \ge 0, \quad \forall k\in K, j\in\Omega^k \tag{3.14}$$

$$y_s^e \in \{0,1\}, \quad \forall e\in E, s\in S^e. \tag{3.15}$$

The objective function (3.9) minimizes the total net cost that is computed as the sum of the service costs and the commodity-related costs (unloading, loading, and transshipment costs) minus the total revenue for the transported commodities. Constraints (3.10) limit the number of containers that can be transported for each demand. Constraints (3.11) ensure that no two services depart from the same port at the same time. Constraints (3.12) link the offered services to the commodity paths. They ensures that, if cargo is transported from one port to another at a particular departure time and with a particular speed (sailing time), then a vessel is assigned to this sailing with a sufficient capacity. Fleet availability is enforced by constraints (3.13). Finally, nonnegativity and binary requirements (3.14)–(3.15) restrict the domain of the variables.

Model (3.9)–(3.15) contains a huge number of variables and a very large number of constraints. Column and row generation can be used again to solve the LP relaxation of (3.14)–(3.15). The service pricing problems can be modeled as a variation of resource constrained shortest path problems. Model (3.9)–(3.15) in fact represents a rich generalization of many service network design models (see e.g. Crainic, 2000). Due to the fundamental change of the master problem, however, not all algorithm components presented in Chapter 2 can be used without modification.

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Chapter 4

Liner Shipping Service Scheduling and Cargo Allocation

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Abstract

Tactical service scheduling and operational cargo allocation are two interdependent problems in liner shipping. The schedules and sailing speeds of individual liner shipping services and the synchronization among all services determine the transit times of containers through a liner shipping network. On the other hand, the market demand in terms of container volume and expected transit times between origin and destination ports drive the schedule design of liner shipping services. We present a graph-based model and a branch-and-price algorithm to solve the combined problem. The goal is to minimize the difference between fuel consumption costs and revenues from transporting containers under consideration of transit time limits. Fuel consumption is modeled as a function of both speed and payload. Results are presented for four liner shipping networks and emphasize the importance of explicitly modeling schedules in large networks; transit times may be severely miscalculated otherwise. The results further show that neglecting payload in the fuel consumption function can result in suboptimal service schedules and cargo routing decisions.

4.1 Introduction

Despite historically low fuel oil prices, the liner shipping industry is struggling with decreasing profitability; for the year 2016 the majority of large liner shipping companies, including market leaders like Danish Maersk Line and French CMA CGM, reported losses. With the South Korean Hanjin Shipping company, one of the ten largest liner shipping carriers went out of business. Historically low freight rates caused by substantial overcapacity and global trade slowdown were the main drivers of the industry's low profitability. Rather than being a temporary phenomenon, the current market situation is expected to become the ”new normal” (Egloff et al., 2016).

In this study, we simultaneously address two problems that affect operational costs as well as the service level of a liner shipping company: The service scheduling problem and the cargo allocation problem. We further explicitly consider service level requirements and model vessel fuel consumption as a function of speed and payload.

The service scheduling problem aims at determining optimal port call times of liner shipping services. The problem considers the required minimum speed and related fuel consumption between two subsequent port calls, i.e. it is a generalization of the simpler speed optimization problem. Besides determining the speed and fuel cost, a schedule of liner shipping services also establishes the transshipment times between two different services that call the same port (Figure 4.1). Particularly in large global or inter-regional liner shipping networks, a large share of containers is transshipped at least once before reaching their final destination. In the current network of Maersk Line, for example, around half of all containers are transshipped at least once. Transshipment times may thus significantly affect the total transit time of containers.

The second problem we address is the cargo allocation problem (CAP). The CAP tries to answer the question of how to route containers through a liner shipping network between their origin and destination. In global liner shipping networks, there may exist many feasible paths for transporting containers between a given origin-destination pair. These paths may vary in the level of service, i.e. in transit time or the number of required transshipments. Figure 4.2 shows two services operated by Maersk Line. Containers that need to be transported from Cartagena to Rio Grande, for example, can use various different paths. These include either direct paths on one of the two shown services or paths with a transshipment between the two services. Note

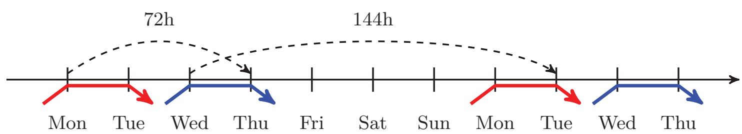
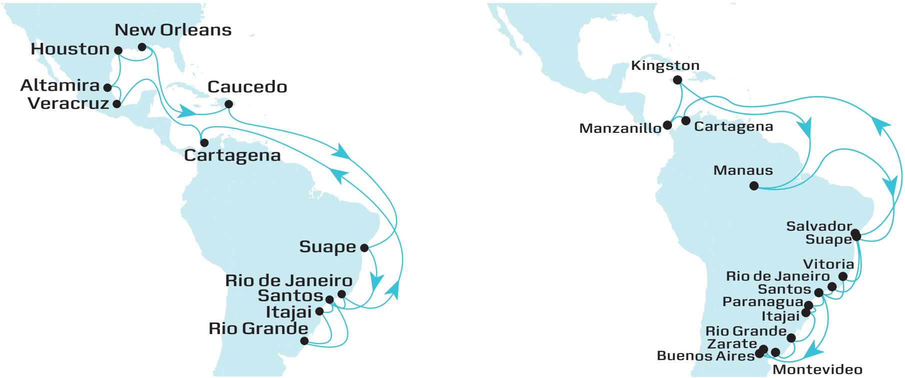


Figure 4.1: Illustration of schedule dependent transshipment times between two services. The example shows the timeline of a port that is called once per week by each of two different services (red and blue arrows). The dashed arrows represent the transshipment time of containers between arrival of the unloading service and the departure of the loading service.



(a) Maersk Line Gulf Express roundtrip

(b) Maersk Line Amazonia Caribbean Express roundtrip

Figure 4.2: Two examples of Maersk Line services (Maersk Line, 2017). Several ports as e.g. Cartagena or Rio de Janeiro are part of both services and allow cargo to be transshipped between the two services.

that containers can be transshipped between the two services at different ports, e.g. in Suape or Rio de Janeiro. From the liner shipping company's point of view, different feasible container paths allow optimizing capacity utilization among different services and service legs.

In this paper we solve the service scheduling and the cargo allocation problem simultaneously to account for the dependencies between both problems. The cargo allocation problem requires the service schedules and the transshipment times between connected services as input. The service schedule design is driven by both fuel consumption costs as well as the need for offering fast transportation between origin and destination ports. The objective is to minimize total fuel consumption costs minus total revenues from transporting containers while satisfying transit time requirements for each demand. We call the combined problem the *liner shipping service scheduling and cargo allocation problem* (LSSCAP). We present a graph-based model formulation of the LSSCAP and describe a branch-and-price solution algorithm with an integrated primal heuristic. The method quickly generates feasible solutions, but can also be used to solve the problem to optimality if time allows.

The second contribution of this work lies in the explicit consideration of the vessels' payload in the fuel consumption function. The large majority of studies that address ship routing or scheduling problems assume fuel consumption to depend only on speed. In practice, however, many other factors affect a vessel's fuel consumption, as for example weather conditions, the vessel's hull condition or its payload. Particularly neglecting the payload may cause substantial fuel consumption estimation errors for vessel types whose load factor typically varies; Psaraftis and Kontovas (2016) present data for two very large crude carriers, whose fuel consumption differs by $25\% - 30\%$ between laden and ballast condition. Whereas container vessels are seldom completely empty, load factors traditionally vary significantly between headhaul and backhaul trades on some trades lanes; on the Asia-Europe, transpacific and transatlantic lanes the headhaul trade was 2.2 times the backhaul trade in 2015 (Waters, 2016). Our optimization model takes a vessel's payload and its impact on fuel consumption explicitly into consideration.

Finally, we use the model and method to assess widely used assumptions and their implications. We show that the choice of a less accurate speed-only dependent fuel consumption function does not only provide wrong estimates of fuel consumption cost, but can also result in the design of suboptimal schedules and cargo allocation plans. We further demonstrate that the common approximation of transshipment times can cause significant underestimation of cargo transit times.

The paper is organized as follows. In Section 4.2 we review literature that addresses service scheduling and cargo allocation problems in liner shipping. We briefly discuss selected contributions related to the issue of neglecting payload in fuel consumption functions. In Section 4.3 we first describe the payload dependent fuel consumption function and its approximation used in our model. We derive fuel consumption function parameters for all LINER-LIB vessel classes and assess the approximation error. The LSSCAP is formally defined in Section 4.4. In Section 4.5 we formulate a graph-based model for the LSSCAP. A branch-and-price algorithm together with an integrated primal heuristic to solve the problem are presented in Section 4.6. In Section 4.7 we report and analyze computational results and conclude in Section 4.8.

4.2 Literature review

Both the liner shipping cargo allocation problem and the service scheduling problem have previously been addressed in the literature. The cargo allocation problem is an operational problem, whereas service scheduling (and, closely related, speed optimization) fall within tactical decision making (see e.g. Meng et al., 2014). Our review focuses on models and solution approaches for these two problems within liner shipping networks that consist of multiple services. For broader literature reviews that cover a wide range of optimization problems within liner shipping, we refer the reader to the surveys by Christiansen et al. (2013) and Meng et al. (2014).

The classical cargo allocation problem or cargo routing problem aims at finding origin-destination cargo paths in a fixed network of services such that vessel capacities are not exceeded. Strictly speaking, it is an integer multi-commodity flow problem, but the integrality constraints are commonly relaxed and any number of fractional containers is rounded, if necessary. Brouer et al. (2011) show that the resulting rounding errors are likely to be negligible in practice, particularly if the number of containers is large. The same authors present a delayed column generation algorithm that solves large instances of the cargo allocation problem with empty container repositioning. The subproblems are shortest path problems and solved efficiently using Dijkstra's algorithm. Although additional constraints as e.g. transit time limits on cargo paths generally make the subproblems hard in terms of computational complexity, the problems can still be solved in very reasonable computational times by label setting algorithms even for large practical liner shipping network instances (Karsten et al., 2015).

Guericke and Tierney (2015) extend the cargo allocation problem by additionally considering sailing speed as a decision variable. For each origin-destination demand a transit time limit has to be satisfied, otherwise it has to be rejected and no revenue is earned for that particular demand. They further consider the repositioning of empty containers. Cargo can be transshipped between services, but the transshipment time is approximated by a constant because services are not scheduled. The work by Karsten et al. (2016) considers a similar problem of simultaneous sailing speed optimization and container routing under consideration of transit time limits. They further consider a scenario in which the number of vessels deployed on a service is also subject to optimization. As in Guericke and Tierney (2015), transshipment times are approximated by a constant.

Reinhardt et al. (2016) evaluate potential fuel cost savings by rescheduling a set of existing services. The cargo routing, however, is assumed to be predetermined and fixed. Transit time limits for the fixed cargo paths have to be respected by the rescheduled network, though. A very similar problem is considered by Wang and Meng (2012a); as in Reinhardt et al. (2016), the container paths through the network are fixed, transit times have to be satisfied and fuel consumption is minimized through sailing speed optimization. Additionally, the authors assume the port stay time at each port to be a random variable and require a sailing time buffer for each sailing leg as a precaution for possible delays.

Wang and Meng (2011) present a combined schedule design and container routing problem. The cost function of their model is an increasing function of the total cargo transit time that shall reflect the inventory cost of cargo while in transit. Their model, however, does not consider vessel fuel costs, which may vary significantly between schedules due to varying sailing speeds and the strong dependency of bunker fuel consumption on sailing speed. Commonly, higher service levels (i.e. shorter cargo transit times) come at the expense of increased operational costs and have to be considered simultaneously if the objective is to improve overall profitability.

All above-mentioned studies and also most of the work on vessel speed optimization have in common that the fuel consumption function is assumed to be independent of the vessel's payload (see e.g. the survey and taxonomy of Psaraftis and Kontovas, 2013; Psaraftis and Kontovas, 2016). Psaraftis (2017) lists the neglect of payload in vessel fuel consumption functions as one common misconception in the literature on ship routing and scheduling. And indeed, the modified admiralty formula, a basic approximation of a vessel's fuel consumption function used in marine engineering, is a function of speed *and* payload (see e.g. Barrass, 2004; Janić, 2014).

Wang and Meng (2012b) apply linear regression over a set of empirically measured pairs of speed and fuel consumption to calibrate a solely speed dependent fuel consumption function for container vessels. The results show that the fuel consumption of the same vessel can actually vary between sailing legs. The results thus at least suggest that other relevant factors than speed must exist.

Noteworthy exceptions among the literature are the papers by Xia et al. (2015) and Wen et al. (2017) that both consider payload in the context of a routing problem involving multiple routes and vessels. To the best of our knowledge, the paper of Xia et al. (2015) is the first to consider a payload dependent fuel consumption function in a liner shipping network context. The authors approximate the fuel consumption function to keep their model linear and correct resulting errors in an iterative procedure embedded in their algorithm. In this work, we present an alternative, but more accurate approximation of the payload dependent fuel consumption function, which we incorporate in a linear model.

4.3 Payload dependent fuel consumption function

We first introduce the payload dependent fuel consumption function below. In Section 4.3.1 we estimate fuel consumption function parameters for all vessel classes of the publicly available LINER-LIB data instances. We describe the derivation of the parameters in detail to encourage other users of the LINER-LIB instances to use the more realistic payload dependent fuel consumption function. In Section 4.3.2 we present a more tractable approximation of the payload dependent fuel consumption function that we will use in our model. We assess the approximation error for the LINER-LIB vessel classes and show that it is negligible for practical applications.

A simple but common approximation of the fuel consumption function of a vessel class e is given by equation

$$f_e(v) := F_e \cdot v^3 \tag{4.1}$$

with variable v denoting the vessel's speed and $F_e > 0$ being a vessel class specific input parameter. Indeed, cubic fuel consumption functions are the most common ones used in the literature (Psaraftis and Kontovas, 2013).

A more realistic approximation of the fuel consumption function that considers both speed and payload is the modified admiralty formula (see e.g. Barrass, 2004; Janić, 2014):

$$f_e(v, \Delta_e) := \hat{F}_e \cdot v^n \cdot \Delta_e^{2/3} \tag{4.2}$$

Δ_e denotes the *displacement* of a vessel of class e and $n \geq 3$ and $\hat{F}_e > 0$ are parameters. Displacement is defined as the weight of the volume of water the ship displaces. It equals the sum of the ship's lightweight (lwt) and actual deadweight (dwt). The lightweight defines the weight of the ship when completely empty and deadweight represents the actual weight of everything the ship carries. The deadweight corresponds to the sum of the weight of loaded cargo, fuel, ballast water, fresh water, provisions and crew.

	LINER-LIB data						Own estimates				
Vessel class	U_e	v_e^-	v_e^+	v_e^*	f_e^*	F_e	lwt_e	owt_e	$\hat{F}_e$	a_e	g_e
Feeder_450	900	10	14	12	18.8	$10.46 \cdot 10^{-3}$	4567	4235	$17.33 \cdot 10^{-6}$	432.5	$2.80 \cdot 10^{-2}$
Feeder_800	1600	10	17	14	23.7	$8.41 \cdot 10^{-3}$	7413	7268	$9.59 \cdot 10^{-6}$	609.3	$2.34 \cdot 10^{-2}$
Panamax_1200	2400	12	19	18	52.5	$9.00 \cdot 10^{-3}$	11346	10486	$7.66 \cdot 10^{-6}$	794.0	$2.05 \cdot 10^{-2}$
Panamax_2400	4800	12	22	16	57.4	$14.01 \cdot 10^{-3}$	19825	17073	$7.96 \cdot 10^{-6}$	1132.2	$1.69 \cdot 10^{-2}$
Post_panamax	8400	12	23	16.5	82.2	$18.30 \cdot 10^{-3}$	33079	21674	$7.55 \cdot 10^{-6}$	1482.2	$1.45 \cdot 10^{-2}$
Super_panamax	15000	12	22	17	126.9	$25.83 \cdot 10^{-3}$	53972	17640	$7.90 \cdot 10^{-6}$	1801.3	$1.26 \cdot 10^{-2}$

Table 4.1: Vessel classes from LINER-LIB and related parameters. The first six columns denote for each vessel class the capacity (note: in TEU), minimum speed, maximum speed, design speed, fuel consumption at design speed and parameter F_e. The remaining five columns report parameter estimates for the payload dependent fuel consumption function.

In the following we use the definition $\Delta_e(w) := lwt_e + owt_e + w$ that defines the displacement of a vessel of class $e \in E$ as a function of the weight w of all loaded containers. lwt_e and owt_e are vessel class dependent constants that represent the vessels' lightweight and the average weight of fuel, ballast water, fresh water, provision and crew, respectively. Assuming owt_e to be constant is a simplifying, though acceptable assumption, because the variation in weight of fuel and ballast water is usually small compared to a vessel's total deadweight. The payload dependent fuel consumption function (4.2) of a vessel class e can thus be reformulated as a function of speed v and the weight w of loaded containers:

$$f_e(v, w) := \hat{F}_e \cdot v^n \cdot (lwt_e + owt_e + w)^{2/3} \tag{4.3}$$

4.3.1 Payload dependent fuel consumption functions for LINER-LIB instances

The LINER-LIB specifies a set of six representative vessel classes of different capacities. Although the LINER-LIB benchmark suite itself does not specify a particular fuel consumption function, Brouer et al. (2014) use a speed-only dependent fuel consumption function in their presented base model. As far as we are aware, the suggested speed-only dependent fuel consumption function has been used in all other studies that use LINER-LIB data instances.

For each vessel class $e \in E$ a design speed v_e^* and the fuel consumption at design speed f_e^* (tons per day, assuming an average load factor) are given. Brouer et al. (2014) define the fuel consumption of a vessel class $e \in E$ at a speed v as $f_e(v) := v^3/v_e^{*3} \cdot f_e^*$. The fuel consumption function can be rewritten in the common form of equation (4.1) as $f_e(v) := F_e \cdot v^3$ with $F_e := f_e^*/v_e^{*3}$.

In order to estimate the parameters of speed and payload dependent fuel consumption function (4.3) for each vessel class of the LINER-LIB, we make the assumption that the simple, speed dependent fuel consumption function $f_e(v) := F_e \cdot v^n$ with $n = 3$ and parameters F_e as defined by Brouer et al. (2014) are correct given an average load factor. In order to calculate the average cargo weight, we assume an average load factor of 0.7 (see e.g. CCWG, 2014; Psaraftis and Kontovas, 2009) and an average gross weight per container of 11 tons (Xia et al., 2015, report an average gross weight of 8-13 tons per container for a large shipping company). The resulting average cargo weight per vessel class is $\bar{w}_e := 0.7 \cdot U_e \cdot 11$ for vessel class $e \in E$, with U_e denoting the capacity of vessel class e in twenty-foot equivalent units (TEU).

By setting both equations (4.1) and (4.2) equal to each other and solving for $\hat{F}_e$, we obtain a coefficient value for each vessel class $e \in E$:

$$\begin{aligned} & F_e \cdot v^3 = \hat{F}_e \cdot v^3 \cdot (lwt_e + owt_e + \bar{w}_e)^{2/3} \\ \Leftrightarrow \quad & \hat{F}_e = \frac{F_e}{(lwt_e + owt_e + \bar{w}_e)^{2/3}} \end{aligned}$$

The calculated coefficients $\hat{F}_e$ for each vessel class are reported in Table 4.1. The derivation of estimates lwt_e and owt_e for all LINER-LIB vessel classes $e \in E$ is based on the vessels' capacity in TEU and described in detail in Section 4.A.

The estimation of the payload dependent fuel consumption function based on the original, simple fuel consumption function has two advantages: First, we preserve each vessel class' characteristics as defined by the original LINER-LIB fuel consumption function. In particular, the relative fuel consumption costs between vessel classes remains the same. The smallest feeder vessel class, for example, has a higher fuel consumption than the `Feeder_800` and even the `Panamax_1200` vessel class for any given speed, as the comparison of the parameter F reveals. Second, the fuel consumption function $f_e(v, w)$ at a load factor of 0.7 and an average container weight of 11 tons corresponds to the payload independent fuel consumption function $f_e(v)$ for all vessel classes. Both functions can thus be directly compared.

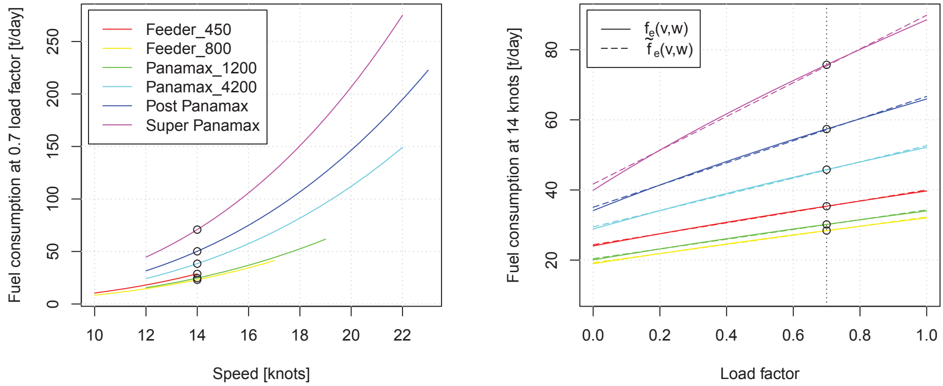
4.3.2 Linear approximation of the payload dependent term

We propose to replace the nonlinear term $(lwt_e + owt_e + w)^{2/3}$ of fuel consumption function (4.3) by a linear approximation $a_e + g_e \cdot w$, with a_e and g_e denoting parameters defined for each vessel class $e \in E$. The resulting approximation of the payload dependent fuel consumption function is

$$\tilde{f}_e(v, w) := \hat{F}_e v^n (a_e + g_e w) \tag{4.4}$$

The approximation is still nonlinear in the speed variable and also because of the multiplication of the two terms, but it becomes linear for any fixed speed.

We will assess the error resulting from the approximation on the LINER-LIB vessel classes. In order to determine parameters a_e and g_e for each vessel class $e \in E$, we first generate sample points from function $(lwt_e + owt_e + w)^{2/3}$ based on a set of evenly distributed cargo weights in the range $w \in [0.0; 11 \cdot U_e]$. Second, we use linear regression to determine the parameters a_e and g_e of the approximation $a_e + g_e \cdot w$ that best fit the sampled set of data points. The



(a) Fuel consumption for a fixed load factor of 0.7 and varying speeds.

(b) Fuel consumption (and linear approximation) for a fixed speed of 14 knots and varying load factors.

Figure 4.3: Fuel consumption functions. The circles in both figures show equivalent fuel consumption points at a speed of 14 knots and a load factor of 0.7.

estimated intercept and coefficient for each vessel class are reported in Table 4.1. Both functions are plotted for each vessel class in Figure 4.3b.

The adjusted R^2 statistic is greater than 0.99 for all vessel classes and the largest error among all vessel classes is $\max_{e \in E, w \in [0,11 \cdot U_e]} |\tilde{f}(v, \tilde{w}) - f(v, w)| = 4.46\%$. The largest errors occur for load factors close to 0.0, i.e. in the case a container ship is completely empty, which in practice is very unlikely to happen during normal operations. If load factors below 0.1 are excluded, the maximum error reduces to 1.48% – a negligible value in the light of all other uncertainties inherent to fuel consumption estimation.

Xia et al. (2015) present an approximation that separates the speed and the payload dependent term. It is of the form $\omega'' + \omega' v^2 + \omega w$, with ω'', ω' and ω denoting linear coefficients, and defined as consumption per distance, not per time. This approximation is simpler than equation (4.4), but less accurate; for a vessel of 10000 TEU capacity and load factors between 0.2 and 1.0, the authors report approximation errors of 7.8% on average, but as large as 30.3% in the worst case.

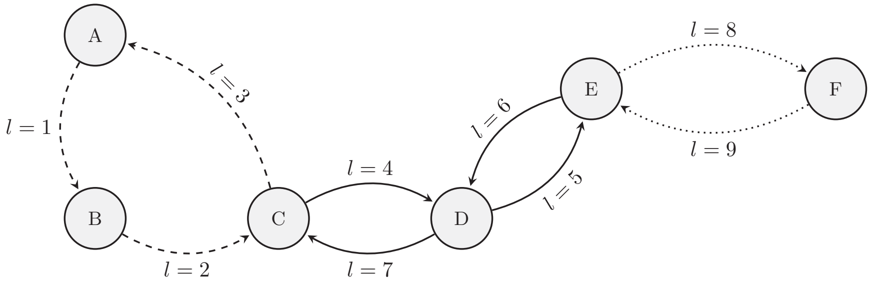


Figure 4.4: Illustration of three different services. Service 1 (dashed) connects to service 2 (solid) and vice versa at port C. Service 2 further connects to Service 3 (dotted) and vice versa at port F. Service 1 and Service 3 are simple services, whereas Service 2 is a butterfly type service calling port D twice.

4.4 Problem statement

We consider a given set of (non-scheduled) *services* S. Each service $s \in S$ is defined by a sequence of port calls that define the sailing route of the service. We use the index set $l \in L_s$ to denote the port calls on service $s \in S$. Multiple port calls for the same port $p \in P$ may exist, as e.g. in butterfly or conveyor belt type routes. We use notation $p(l)$ to denote the port $p \in P$ that corresponds to port call $l \in L$. Figure 4.4 shows a small network consisting of two simple services $s = 1$ and $s = 3$ and one butterfly service $s = 2$. Note that service $s = 2$ visits three ports but consists of $|L_2| = 4$ port calls, with port D being called twice. Set $L = \cup_{s \in S} L_s$ denotes the index set of all port calls in the considered liner shipping network. The number of sailing legs on each service equals the number of port calls and there is a one-to-one correspondence between a port call and a sailing leg; in the remainder of this paper we therefore use set $l \in L_s$ to also denote the sailing leg on service $s \in S$ that follows port call l.

Each service $s \in S$ is operated by a given number and type of vessels. We assume a weekly service frequency for all services, as it is predominant in the liner shipping industry. Thus, the number of vessels N^s that operate a service also define the total round-trip time of a service in weeks; a service of a duration of n weeks requires exactly n vessels to ensure a weekly port call frequency. U^s denotes the capacity of the vessel type deployed on service $s \in S$.

Set K represents the set of all origin-destination demands. Each demand is defined by its origin and destination port, a weekly quantity of containers q_k, an average weight w_k per container and a revenue r_k earned per container if transported. Furthermore, a transit time limit T_k defining a maximum duration between pick-up and delivery is given for every demand.

The goal is to simultaneously find a feasible schedule for each service $s \in S$ and to allocate cargo to the services such that the total fuel consumption cost minus the revenues from transporting cargo are minimized. A schedule is feasible if the total round trip time (in weeks) equals the deployed number of vessels and if the speed restrictions of the deployed vessel class are satisfied on each sailing leg. A cargo path is only feasible, if the total transit time including transshipment times at ports (if applicable) does not exceed the maximum transit time T_k of the corresponding demand $k \in K$. The cargo allocation also affects the fuel consumption of vessels, as detailed in Section 4.3.

4.5 Model formulation

We model the LSSCAP over a time-space graph that is described in detail in Section 4.5.1. In Section 4.5.2 we present a linear mixed integer programming formulation of the problem. It uses the approximated payload dependent fuel consumption function presented in Section 4.3.2.

4.5.1 Time-space graph

We model the problem over a time-space graph $G = (V, A)$, in which each port call $l \in L$ is represented by a set of vertices V_l that correspond to different feasible port call times during a week. The set of feasible port call times may differ between ports, reflecting port operating policies or availabilities, for example. For simplicity, we assume a common set H of discretized weekly berth times for all port calls in the remainder of the paper. The weekly time of 168 hours is discretized into uniform steps of length $\bar{h}$ (required to be a divisor of 168 hours). The size of set H depends on the time discretization parameter $\bar{h}$ and is defined as $H = \{0, 1 \cdot \bar{h}, 2 \cdot \bar{h}, ..., 168 - \bar{h}\}$. The number of vertices equals the product of the number of port calls and the number of time steps, $|V| = |L| \times |H|$. Figure 4.5 illustrates the time-space graph that corresponds to the liner shipping network shown in Figure 4.4.

The set A of arcs can be divided into two disjoint sets, sailing arcs A^S and transshipment arcs A^T. The set A^S of sailing arcs denotes feasible sailings between two subsequent port calls, including a constant port stay time at the port of the destination port call. Hence, each arc $a \in A^S$ is associated with a particular sailing leg $l \in L$ and we denote the set of arcs corresponding to sailing leg $l \in L$ by A_l^S, with $\cup_{l \in L} A_l^S = A^S$. An arc $a \in A_l^S$ is feasible if the associated sailing time lies within the vessel's minimum and maximum sailing time for sailing leg l. The duration t_a of an arc $a \in A^S$ is allowed to exceed a week and for very long distance legs parallel arcs may exist, representing sailing times that differ by one week. Thus, the duration t_a of any feasible sailing arc $a = (u, v)$ between vertices $u = (l_1, h_1)$ and $v = (l_2, h_2)$ satisfies the equation $t_a = h_2 - h_1 + \tau_a \cdot 168$, with τ_a denoting a non-negative integer constant. The port stay time associated with an arc reflects the port stay time at the port of the arc's destination port call. We assume a constant port stay time for each port call; this value should reflect the expected port stay time for the corresponding service at the corresponding port. The cost c_a of arc $a \in A^S$ represents a vessel's fuel consumption cost relative to the distance and sailing speed associated with the arc. A precise definition will be given below.

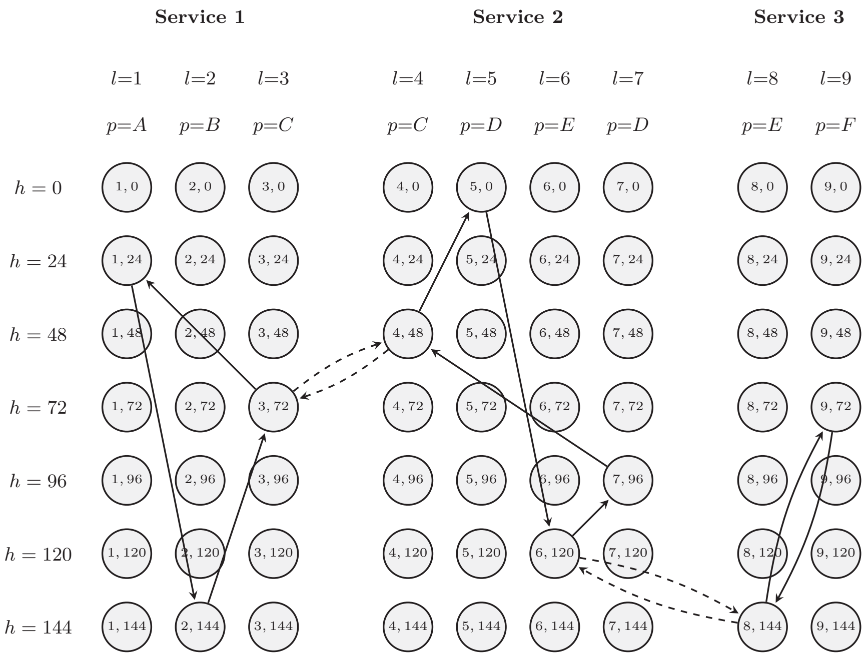


Figure 4.5: Time-space graph $G = (V, A)$ with a time discretization of $\bar{h} = 24$ corresponding to the liner shipping network shown in Figure 4.4. The displayed arcs represent one feasible solution to the scheduling problem. The solid arcs represent scheduled sailings, whereas the dashed arcs represent transshipments of containers between services.

The set A^T of transshipment arcs represents transshipments of containers between services at a common port. Arcs $a \in A^T$ may only connect two different vertices $u = (l_1, h_1)$ and $v = (l_2, h_2)$, if the port calls are different but the called port is the same, i.e. if $l_1 \neq l_2$ and $p(l_1) = p(l_2)$. Transshipment arcs represent movement in time only. The actual transshipment time depends on the schedules of the two corresponding services and on a minimum transshipment time T^{TS}. The minimum transshipment time defines the time that is usually required for unloading, handling and loading of transshipped containers at the port. Given a weekly service frequency, a container has to wait at most $T^{TS} + 168$ hours. As an example, assume a port stay time of 24 hours (included in the duration of the incoming sailing arc) and consider vertices $(4, 48)$ and $(3, 72)$ in Figure 4.5; the transshipment time from vertex $(4, 48)$ to vertex $(3, 72)$ equals $72 - 48 + 24 = 48$ hours, if the minimum transshipment time T^{TS} is less or equal to 48 hours, and $72 - 48 + 24 + 168 = 216$ hours if the minimum transshipment time is required to be larger than 48 hours. The arc duration t_a of arc $a \in A^T$ of the given example would equal 24 or 192 hours, respectively (not including the port stay time). In case hot berthing, i.e. the simultaneous exchange of containers between two vessels, is feasible at a port, the arc duration between two vertices (l_1, h_1) and (l_2, h_2) with $h_1 = h_2$ is set to $t_a = 0$. The cost c_a of transshipment arc

$a \in A^T$ equals the unit transshipment cost of the associated port.

4.5.2 Mathematical model

We formulate the LSSCAP as a MIP model over graph $G = (V, A)$. We use two sets of decision variables in the model. The first set consists of binary variables y_a that are defined over the set of sailing arcs A^S. The variable indicates whether a particular arc is used ($y_a = 1$) or not ($y_a = 0$). As every arc corresponds to a sailing leg $l \in L_s$ of a particular service $s \in S$, the decision variable thus defines whether this sailing leg is sailed at a particular time and with a particular speed.

The second set of continuous decision variables x_j^k denotes the number of containers of demand $k \in K$ that are transported along path $j \in \Omega^k$, with Ω^k denoting the set of all feasible paths of demand $k \in K$. Relaxing the integrality requirement on the number of containers transported along a path is a simplification that is virtually negligible in practice, particularly against the background of demands generally being estimates. Brouer et al. (2011) show that solutions to the CAP are often integer and if they are not, the optimality gap is minimal ($< 0.01\%$).

We use binary parameters b_{ja} to indicate if a container path $j \in \Omega^k, k \in K$ uses arc $a \in A^S$ ($b_{ja} = 1$) or not ($b_{ja} = 0$). The cost parameter c_j^k represents the cargo handling cost along path $j \in \Omega^k$. It equals the sum of the cost for loading a container at its origin port O^k, the cost for unloading at the destination port D^k and the unit cost for transshipments, if the path includes any. We further use a parameter p^k that denotes the penalty for rejecting one unit of demand $k \in K$. Although charging a penalty for not transporting a container is equivalent to increasing the revenue for transporting a container plus adding a constant to the objective function value, we explicitly state the penalty parameter for easier comparison of computational results with previous studies using LINER-LIB data instances.

To derive the fuel cost function, we first observe that for any given sailing arc $a \in A^S$ the sailing speed is fixed and thus a parameter $\bar{v}_a$. Let t_a^S and t_a^P (with $t_a^S + t_a^P = t_a$) denote the sailing and port stay time (in hours), respectively, associated with sailing arc $a \in A^S$. Let further $e(a)$ denote the vessel class that operates the service that is associated with arc $a \in A^S$. The approximated payload dependent *daily* fuel consumption function (4.4) introduced in Section 4.3 can then be used to formulate the fuel consumption associated with the sailing time of arc $a \in A^S$ as a function $f_a(\boldsymbol{x})$ of the cargo allocation $\boldsymbol{x} := \{x_j^k\}_{k \in K, j \in \Omega^k}$:

$$\begin{aligned} f_a(\boldsymbol{x}) :&= (t_a^S/24)\hat{F}_{e(a)}\bar{v}_a^n \left(a_{e(a)} + g_{e(a)} \left(\sum_{k \in K} \sum_{j \in \Omega^k} b_{ja}\bar{w}x_j^k \right) \right) \\ &= (t_a^S/24)\hat{F}_{e(a)}\bar{v}_a^n a_{e(a)} + (t_a^S/24)\hat{F}_{e(a)}\bar{v}_a^n g_{e(a)} \left(\sum_{k \in K} \sum_{j \in \Omega^k} b_{ja}\bar{w}x_j^k \right) \end{aligned} \tag{4.5}$$

where term $\sum_{k \in K} \sum_{j \in \Omega^k} b_{ja} w^k x_j^k$ represents the weight of all containers transported along arc

$a \in A^S$. Note that the sailing time t_a^S (in hours) is divided by 24 to obtain the equivalent time in days.

The fuel consumption associated with the port stay time of arc $a \in A^S$ depends only on the idling time t_a^P and thus is a constant for each arc $a \in A^S$. We denote it by f_a^P. Parameter p^F represents the fuel price. The total fuel consumption cost can be expressed by the term $\sum_{a \in A^S} y_a p^F \left(f_a(\boldsymbol{x}) + f_a^P \right)$.

The complete objective function, with $f_a(\boldsymbol{x})$ being replaced by the full expression, is

$$\begin{aligned}\min \quad & \sum_{a \in A^S} p^F \left((t_a^S/24) \hat{F} \bar{v}_a^n a_{e(a)} y_a + (t_a^S/24) \hat{F} \bar{v}_a^n g_{e(a)} y_a \left(\sum_{k \in K} \sum_{j \in \Omega^k} b_{ja} \bar{w} x_j^k \right) + f_a^P y_a \right) \\ & + \sum_{k \in K} \sum_{j \in \Omega^k} c_j^k x_j^k + \sum_{k \in K} \left(q^k - \sum_{j \in \Omega^k} x_j^k \right) p^k - \sum_{k \in K} \sum_{j \in \Omega^k} x_j^k z^k \end{aligned} \tag{4.6}$$

The first expression of the objective function represents the total fuel consumption, multiplied by the fuel price p^F. It consists of three terms. The first term represents the vessels' fuel consumption cost for sailing empty, i.e. without any payload. The second term stands for the fuel consumption caused by the additional weight of transported containers. The first two terms thus represent the fuel consumption while sailing. The fuel consumption while idling at the destination port of arc $a \in A^S$ is represented by the third term. The remaining expressions describe the cargo handling cost, the demand rejection penalties and the revenue from transporting demand, respectively.

Note that the objective function is nonlinear in its current form due to the multiplication of decision variables in the second expression of the first term. However, the binary y_a variable in the second term of the fuel consumption expression is redundant because by definition the cargo transported along an arc is $b_{ja} x_j^k = 0$ for any $k \in K, j \in \Omega^k$ if that arc is not operated, i.e. if $y_a = 0$. Thus, variable y_a can be omitted from the second expression of the fuel consumption term.

The resulting objective function is linear and can be rewritten as

$$\min \quad \sum_{a \in A^S} c_a y_a + \sum_{k \in K} \sum_{j \in \Omega^k} \tilde{c}_j^k x_j^k + \sum_{k \in K} \left(q^k - \sum_{j \in \Omega^k} x_j^k \right) p^k - \sum_{k \in K} \sum_{j \in \Omega^k} r^k x_j^k \tag{4.7}$$

with constants c_a and $\tilde{c}_j^k$ being defined as

$$c_a := p^F \cdot (t_a^S/24) \hat{F} \bar{v}_a^n a_{e(a)} + p^F f_a^P \tag{4.8}$$

$$\tilde{c}_j^k := c_j^k + p^F \sum_{a \in A^S} (t_a^S/24) \hat{F} \bar{v}_a^n g_{e(a)} b_{ja} \bar{w} \tag{4.9}$$

Parameter c_a represents the fuel consumption cost of vessel class $e(a)$ for sailing empty along arc $a \in A^S$ plus the fuel consumption cost while idling at the destination port of arc $a \in A^S$.

Parameter $\tilde{c}_j^k$, on the other hand, denotes the sum of the handling cost (c_j^k) and the weight related fuel consumption cost caused by one container of demand $k \in K$ along path $j \in \Omega^k$.

Problem LSSCAP can then be formulated as a linear MIP model:

$$\min \quad \sum_{a \in A^S} c_a y_a + \sum_{k \in K} \sum_{j \in \Omega^k} \tilde{c}_j^k x_j^k + \sum_{k \in K} \left(q^k - \sum_{j \in \Omega^k} x_j^k \right) p^k - \sum_{k \in K} \sum_{j \in \Omega^k} r^k x_j^k \tag{4.10}$$

$$\text{s.t.} \quad \sum_{j \in \Omega^k} x_j^k \leq q^k, \qquad \forall k \in K \qquad [\alpha] \tag{4.11}$$

$$\sum_{a \in A_l^S} y_a = 1, \qquad \forall s \in S, l \in L_s \qquad [\gamma] \tag{4.12}$$

$$\sum_{a \in A_+^S(v)} y_a - \sum_{a \in A_-^S(v)} y_a = 0, \qquad \forall s \in S, v \in V_s, \qquad [\delta] \tag{4.13}$$

$$\sum_{k \in K} \sum_{j \in \Omega^k} b_{ja} x_j^k - U^s y_a \leq 0, \qquad \forall s \in S, l \in L_s, a \in A_l^S \qquad [\lambda] \tag{4.14}$$

$$\sum_{l \in L_s} \sum_{a \in A_l^S} t_a y_a = 168 N^s, \qquad \forall s \in S \qquad [\pi] \tag{4.15}$$

$$x_j^k \geq 0, \qquad \forall k \in K, j \in \Omega^k \tag{4.16}$$

$$y_a \in \{0, 1\}, \qquad \forall s \in S, l \in L_s, a \in A_l^S \tag{4.17}$$

Constraints (4.11) limit the number of containers that can be transported for each demand. Constraints (4.12) require each port call (leg, resp.) to be scheduled exactly once. That is, each leg has to be sailed at exactly one particular time and with one particular speed. The constraints represent *generalized upper bound* (GUB) constraints. Constraints (4.13) are flow conservation constraints. The arc subsets $A_-^S(v)$ and $A_+^S(v)$ denote the incoming and outgoing sailing arcs of vertex v. The constraints require that if a vessel arrives at a port at a particular time, it has to leave the same port directly after finishing cargo handling. Constraints (4.14) ensure that containers can only be transported along an arc $a \in A^S$ if a vessel with sufficient remaining capacity operates the corresponding sailing. These constraints are commonly called linking constraints, as they link the capacity consuming commodity path variables x_j^k to the capacity deploying sailing arc variables y_a. Constraints (4.15) require the duration of service $s \in S$ (in weeks) to be equal to the number of vessels N^s deployed on that service. Finally, constraints (4.16)–(4.17) define the domain of the decision variables. $\boldsymbol{\alpha}, \boldsymbol{\gamma}, \boldsymbol{\delta}, \boldsymbol{\lambda}$ and $\boldsymbol{\pi}$ denote the dual variable vectors of the linear program that results from relaxing the integrality constraints (4.17).

4.6 A branch-and-price algorithm for the LSSCAP

Branch-and-price combines the well-known concepts of branch-and-bound to solve integer or mixed integer programming problems and column generation to solve linear programming (LP) problems. In simple terms, branch-and-price extends the branch-and-bound algorithm paradigm by applying column generation to solve the LP problems that correspond to the nodes of the branch-and-bound tree. Whereas the theoretical concept is rather straightforward, various design questions and implementation issues commonly arise when branch-and-price is implemented for practical problems. Nemhauser and Wolsey (1999) provide an introduction to the branch-and-bound algorithm paradigm and address the most common design questions. Barnhart et al. (1998) give an introduction to branch-and-price and review issues that may arise when implementing branch-and-price algorithms.

Column generation is a solution technique that starts with solving a smaller version of the original LP problem by only considering a subset of the decision variables. It iteratively identifies and adds new variables to the model and resolves it until no further improvement is possible. The generation of new variables is termed pricing and often requires the solution of a pricing problem. Ideally, only a small subset of variables has to be generated until optimality can be proved. The book on column generation by Desaulniers et al. (2005) discusses a wide range of column generation algorithms and applications, many of which in the context of integer programming problems.

The primary goal of most branch-and-price algorithms is to find the *optimal* solution to a combinatorial problem. Branch-and-price algorithm frameworks can, however, also be used to find *good* solutions without proving optimality. We can anticipate that some of the problem instances we address in Section 4.7 are too difficult to be solved to optimality in reasonable time and the goal of finding good feasible solutions becomes our major concern for these instances.

Algorithm 3 describes the branch-and-price framework we use for solving LSSCAP instances. Below we discuss relevant implementation details of the algorithm.

As the set Ω of scheduled cargo paths in model (4.10)-(4.17) is too large to be considered explicitly for realistic liner shipping network instances, we use column generation to dynamically generate and add promising cargo paths. We call the LP relaxation of model (4.10)-(4.17) defined over a restricted set $\Omega' \subset \Omega$ the restricted master problem (RMP). In Section 4.6.1 we describe the implementation of our column generation based solution algorithm to solve the RMPs at the nodes of the branch-and-bound tree. The pricing problems, a variation of resource constrained shortest path problems, are described in detail in Section 4.6.2 together with a fast label setting algorithm and preprocessing techniques. In Section 4.6.3 we describe design decisions for the branch-and-bound framework of Algorithm 3. Algorithm 3 invokes a primal heuristic at each unprocessed node once it is selected for processing. The primal heuristic plays a key role in finding good feasible solutions early and is described in Section 4.6.4.

Algorithm 3 Branch-and-price framework

Initialize $\Phi \leftarrow \emptyset$, $\Omega' \leftarrow \emptyset$, $z(\chi^*) := \infty$
Bound root node ϕ_0 and add to unprocessed nodes: $\Phi := \Phi \cup \{\phi_0\}$
while $\Phi \neq \emptyset$ **do**
  Select node $\phi \in \Phi$, $\Phi := \Phi \backslash \{\phi\}$ ▷ Section 4.6.3
  Branch on node ϕ generating m child nodes $\phi_1, \ldots, \phi_m$ ▷ Section 4.6.3
  for $1 \leq i \leq m$ **do**
    Bound ϕ_i and obtain solution χ_i ▷ Sections 4.6.1 and 4.6.2
    if χ_i is infeasible **then**
      Prune (by infeasibility)
    else if χ_i is fractional **then**
      if $z(\chi_i) \geq z(\chi^*)$ **then**
        Prune (by bound)
      else
        $\Phi := \Phi \cup \{\phi_i\}$
        RunPrimalHeuristic ▷ Section 4.6.4
      end if
    else
      if $z(\chi_i) < z(\chi^*)$ **then**
        $\chi^* \leftarrow \chi_i$
      end if
      Prune (by optimality)
    end if
  end for
end while

4.6.1 Restricted master problem

At each node of the branch-and-bound tree we apply column generation to solve the corresponding RMP. The pricing problems to generate new cargo paths are described in Section 4.6.2. The first RMP has to be solved at the root node of the branch-and-bound tree (Algorithm 3). No initial cargo paths are required to obtain a feasible solution to the RMP, i.e. set Ω' of cargo paths can initially be empty. The column generation algorithm further adds the arc capacity constraints of the RMP dynamically. Whenever a cargo path using the corresponding arc is added for the first time.

The objective function value of the solution to the RMP only constitutes a lower bound for its node, if the RMP is solved to optimality. Solving the RMP to optimality can be expensive, though, because of common column generation pitfalls like a slow convergence, for example (Vanderbeck, 2005). We can, however, calculate a lower bound on the optimal solution of the RMP at each column generation iteration (see e.g. Desrosiers and Lübbecke, 2005). Let $\bar{c}^*_{k(i)}$

denote the most negative reduced cost among all found cargo paths of demand $k \in K$ at iteration i and let $\overline{z}_{(i)}$ denote the objective function value of the RMP at the same iteration i, defining an upper bound on the optimal solution to the RMP. As the amount of containers to be sent along a path $j \in \Omega_k$ cannot exceed q_k units, $\underline{z}_{(i)} = \overline{z}_{(i)} + \sum_{k \in K} \bar{c}^*_{k(i)} q_k$ defines a lower bound on the optimal solution of the RMP at each column generation iteration i. As the lower bound does not necessarily increase at each iteration, we use $\underline{z}^*_{(i)} := \max_{\hat{i} \in \{1,\ldots,i\}} \underline{z}_{(\hat{i})}$ to denote the best known lower bound on the solution of the RMP at iteration i.

We can therefore prematurely stop column generation and still calculate a valid lower bound for the corresponding node. In the final implementation we impose a limit $I^{\max}_{\text{cg}}$ on the number of column generation iterations performed at each node. We further define a threshold gap G_{cg} and stop column generation whenever $(\overline{z}_{(i)} - \underline{z}^*_{(i)})/|\overline{z}_{(i)}| \leq G_{\text{cg}}$. In case the stopping criterion evaluates to true but the current solution to the RMP is integral, the stopping criterion is ignored to not prune the node mistakenly by optimality afterwards. Parameter $N^{\max}_{\text{cg}}$ denotes the maximum number of negative reduced cost cargo paths to be added for each demand $k \in K$ at each column generation iteration.

The branch-and-price algorithm presented in this section can be extended towards a branch-price-and-cut algorithm. We tested the dynamic application of *strong linking* inequalities that are often successfully used to tighten formulations of capacitated network design problems (see e.g. Frangioni and Gendron, 2009). The port call cover constraints (4.12) already enforce capacity deployment, and minor bound improvements per column generation iteration came at the expense of significantly increased LP solution times and a worse overall performance. We thus do not use cutting planes in the final implementation.

4.6.2 Cargo path pricing

The cargo path pricing problem associated with a demand $k \in K$ consists of finding the most negative reduced cost path on graph from an origin port to a destination port while respecting the transit time limit. As a port is generally represented by multiple vertices $v \in V$ in graph $G = (V, A)$, we define an auxiliary graph $\bar{G} = (\bar{V}, \bar{A})$ as follows: Let o_p and d_p denote origin and destination vertices representing port $p \in P$ and let $V^O := \cup_{p \in P}\{o_p\}$ and $V^D := \cup_{p \in P}\{d_p\}$ denote the vertex sets that represent all origin-port and destination-port vertices, respectively.

Similarly, we define auxiliary arc sets A^O and A^D. Set A^O contains all arcs that connect vertices $o_p \in V^O$ with vertices $v = (l, h) \in V$ that are associated with port p, i.e. $p(l) = p$. Set A^D contains all arcs that connect all vertices $v = (l, h) \in V$ to their corresponding destination-port vertex $d_p, p = p(l_1)$. For each arc $a \in A^O$ and $a \in A^D$ we define the arc cost c_a as the unit loading and unit unloading cost, respectively, associated with the corresponding origin and destination ports. The duration of arcs in both sets is $t_a = 0$. The vertex and arc set of the auxiliary graph $\bar{G}$ are thus defined as $\bar{V} := V \cup V^O \cup V^D$ and $\bar{A} := A \cup A^O \cup A^D$.

The cargo path pricing problem can then be modeled as a resource constrained shortest path problem (RCSPP) over auxiliary graph $\bar{G} = (\bar{V}, \bar{A})$, with cost and time as resources. The reduced cost of each arc $a \in \bar{A}$ is defined as

$$\bar{c}_a := \begin{cases} (t_a^S/24)\hat{F}\bar{v}_a^n g_{e(a)}\bar{w}p^F - \lambda_a, & \text{if } a \in \bar{A} \cap A^S \\ \\ c_a, & \text{if } a \in \bar{A} \setminus A^S \end{cases} \tag{4.18a}$$

Note that the reduced cost of sending one container along sailing arc $a \in A^S$ is independent of the demand $k \in K$ of that container, if the average container weight $\bar{w}$ is assumed to not differ between demands. The reduced cost of a particular path $j \in \Omega^k$ equals $\bar{c}_j^k := -r^k - p^k - \alpha_k + \sum_{a \in \bar{A}} b_{ja}\bar{c}_a$.

In case the reduced cost of all arcs is independent of the demand $k \in K$ that uses an arc, the cargo pricing problems can be formulated as one-to-all RCSPPs, resulting in one pricing problem per origin port. Otherwise, a cargo pricing problem has to be solved for each single demand $k \in K$. We will describe the label setting algorithm and all preprocessing techniques for the more general one-to-all RCSPP. The one-to-one RCSPP is a special case of the one-to-all RCSPP. The presented label setting algorithm is an improved version of the label setting algorithm by Karsten et al. (2015). For a broader introduction to variants of RCSPPs and solution algorithms, see Irnich and Desaulniers (2005).

Each RCSPP is associated with an origin port $p \in P$. Let $\bar{K}$ denote the set of demands that originate at port $p \in P$ (we omit index p for this set and all components associated with a particular pricing problem $p \in P$ in this paragraph for clarity). With each label we associate a cost C and a time T representing the reduced cost and time associated with the path of that label. Both resources are constrained; the cost resource is implicitly constrained by the requirement that the reduced cost of a path has to be negative and the time resource is constrained by the transit time limits T_k associated with demands $k \in \bar{K}$. Let $[0, C_v]$ and $[0, T_v]$ denote the resource windows for the cost and time component at vertex v of pricing problem $p \in P$. Cost upper bounds C_v are set to the largest feasible path cost of pricing problem $p \in P$, which is defined as $C_v := \max_{k \in \bar{K}} r^k + p^k + \alpha_k$. The cost upper bounds have to be updated each time the dual variable values α_k change.

The derivation of tight upper bounds T_v on the time resource at vertex $v \in \bar{V}$ can be done iteratively. First, we set the upper bound T_v of all vertices $v \in \bar{V}$ whose associated port $p(v)$ is destination port $d(k)$ of at least one demand $k \in \bar{K}$ to the largest transit time limit among these demands. If ports exist where no demand terminates, the upper bound T_v of vertices corresponding to these ports is set to 0 initially:

$$T_v := \max\left\{0, \max_{k \in \bar{K}: d(k) = p(v)} T_k\right\} \quad \forall v \in \bar{V} \tag{4.19}$$

These bounds represent minimal lower bounds; in fact, they may be too tight and discard feasible labels and thus cargo paths. In a second step, we iteratively loosen these bounds if appropriate.

We iterate through all vertices, always selecting the vertex v^* with the largest upper bound T_v among all vertices not previously selected. We calculate the shortest paths from all other vertices to vertex v^* and update their bounds. Let $t^{\mathrm{SP}}_{uv^*}$ denote the duration of the shortest path from vertex $u \in \bar{V}$ to vertex $v^* \in \bar{V}$. The upper bound on the time resource at vertex u is then updated as

$$T_u := \max\left\{T_u, T_u - t^{\mathrm{SP}}_{uv^*}\right\} \quad \forall u \in \bar{V} \backslash v^* \tag{4.20}$$

After the update, the vertex with the largest (updated) upper bound on the time resource is selected among the vertices not previously selected. Note that the upper bound on the time resource of a selected vertex will never exceed that of any previously selected vertex.

An equivalent iterative procedure can be used to tighten the upper bounds on the cost resource at each vertex. Different to the bounds on the time resource, however, the bounds on the cost resource would have to be updated at each iteration, because they depend on the dual variable values. For this reason we did not implement the tightening procedure for the cost component.

The label setting algorithm processes labels in ascending order of path duration. Due to the used time discretization, labels can be grouped based on their discrete duration and no explicit sorting is necessary to identify the label with the shortest duration. The algorithm uses standard extension functions for both the cost and the time component along arcs and applies a standard dominance rule. At termination, multiple negative reduced cost paths can be returned, if they exist.

4.6.3 Node selection and branching

At each iteration of the branch-and-price algorithm, a node has to be selected for processing. The most common node selection strategies are *depth-first*, *best-first* or combinations of both (see e.g. Nemhauser and Wolsey, 1999). The depth-first strategy prioritizes the exploration of nodes deep in the tree. It is based on the premise that solutions of deep nodes are more likely to be (integer) feasible, as they are more restricted. Finding good upper bounds early in the process may allow to prune other nodes and subtrees by bound before unnecessarily exploring them. The best-first strategy always selects the node with the lowest LB. The rationale behind the strategy is that the node with the best LB bears the largest potential of finding the best feasible solution. The best-first strategy also provides the tightest lower bound for any fixed number of branch-and-price iterations. In our implementation we combine a best-first strategy with a primal heuristic that finds feasible integer solutions at each node of the branch-and-bound tree.

Once the node for branching is chosen, the variable to branch on has to be selected. The only non-continuous variables in the model are the binary service arc variables y_a. Branching on fractional y_a variables is feasible and does not increase the complexity of the cargo path pricing problems. However, the resulting branches are rather unbalanced; the up-branch requires a leg to be sailed at a particular departure time and speed, whereas the down-branch only forbids a single departure time and speed combination. Rather than branching on arcs, Algorithm 3 branches on vertices, which represent pairs of port call and time (l, h); the up-branch fixes the time of a particular port call, whereas the down-branch forbids that time for the same port call.

Given the GUB constraints in form of the port call cover constraints (4.12), we also tested a more balanced GUB branching strategy. We will briefly comment on it in Section 4.7.

The variable for branching is selected in two steps: First, a service $\hat{s} \in S$ is selected. The selected service $\hat{s}$ is the one for which $|L_s| - \eta_s$ is maximal, with η_s denoting the number of branching constraints that fix vertices of service $s \in S$. Vertices on large services are more likely to be selected, and services with a large number of fixed vertices are less likely to be selected. Second, among all vertices of service $\hat{s} \in S$ the algorithm selects the one with the largest fractional vessel deployment. In case of ties, a service or variable is chosen randomly among all best candidates.

4.6.4 Primal heuristic

In order to find good feasible solutions quickly, we apply a primal heuristic throughout the execution of the branch-and-price algorithm. The heuristic is run at each node once it is solved and not (immediately) pruned (line 15). The heuristic consists of two steps; it first generates a feasible schedule $\bar{\mathbf{y}}$ and then allocates cargo to the scheduled services. The schedule is constructed heuristically, but the cargo allocation problem is solved to optimality.

To generate a feasible schedule the heuristic makes use of the following property of model (4.10)–(4.17): if the cargo flow variables x_j^k are removed, constraints (4.11) and (4.14) become redundant and can be removed. We replace the original objective function by a function that maximizes the weighted sum of the sailing arc variables y_a, with w_a denoting the weight assigned to each arc $a \in A_s^S, s \in S$. The resulting problem LSSCAP-UB formulates as

$$\text{LSSCAP-UB}: \qquad \max \qquad \sum_{s\in S}\sum_{l\in L_s}\sum_{a\in A_l^S} w_a y_a \tag{4.21}$$

$$\text{s.t.} \qquad \sum_{a\in A_l^S} y_a = 1, \qquad l \in L_s, s \in S \tag{4.22}$$

$$\sum_{a\in A_+^S(v)} y_a - \sum_{a\in A_-^S(v)} y_a = 0, \qquad v \in V_s, s \in S \tag{4.23}$$

$$\sum_{l\in L_s}\sum_{a\in A_l^S} t_a y_a \le 168N^s, \quad s \in S \tag{4.24}$$

$$y_a \in \{0,1\}, \qquad \forall a \in A_l^S, l \in L_s, s \in S \tag{4.25}$$

Problem (4.21)-(4.25) is a binary integer program (BIP).

The weights w_a associated with sailing arcs $a \in A^S$ serve as a proxy for the estimated importance of sailing the corresponding scheduled leg. All weights w_a are based on the fractional solution $(\mathbf{y}^*, \mathbf{x}^*)$ of the restricted master problem at the active node and defined as the total flow of containers on arc $a \in A$, i.e.

$$w_a := \sum_{k \in K} \sum_{j \in \Omega_k} b_{ja} x_j^{k*} \qquad \forall a \in A \tag{4.26}$$

The choice is based on preliminary tests of different definitions of weights w_a. The resulting BIP is solved to optimality using a commercial MIP solver.

In a second step cargo is allocated to the scheduled services. The service arc variables in the RMP are fixed according to the solution found in the first step of the heuristic, $\mathbf{y} = \bar{\mathbf{y}}$. The cargo allocation problem is then solved by column generation as described in Sections 4.6.1 and 4.6.2, but no limits on the number of iterations or on the number of negative reduced cost columns to add are imposed. The found solution defines the optimal cargo allocation for the fixed schedule.

In case the solution found by the heuristic is not significantly worse than the best known solution, we re-run it $I_{\text{heu}}^{\max} - 1$ times. The heuristic is deterministic and to avoid the same solution being found again, all previously found solutions are forbidden by adding tabu constraints of the form

$$\sum_{a \in A: \bar{y}_a = 1} y_a \leq |L| - 1 \tag{4.27}$$

Once the primal heuristic terminates, all tabu constraints are removed. Cargo paths found by the primal heuristic and previously not contained in the cargo path pool are discarded and not kept in the cargo path pool.

4.7 Computational results

In Section 4.7.1 we first describe the unscheduled liner shipping networks that we used for our computational tests. We have tested different variations of the branch-and-price algorithm as well as different parameter combinations. In Section 4.7.2 we describe the final set of parameters and analyze the performance of the branch-and-price algorithm on different instances. We also briefly share insights obtained during the testing and calibration phase.

Two contributions of this work are the explicit design of schedules and the consideration of payload in the fuel consumption functions. Two research questions arise: In Section 4.7.3 we first analyze if the choice of the fuel consumption functions affects the schedule design and cargo routing decisions. In Section 4.7.4 we assess the value of constructing schedules compared to a pure speed optimization scenario in which transshipment times are approximated.

The branch-and-price framework and the labeling algorithm to solve the pricing problems are implemented in C++. To model the underlying graphs we used the Lemon graph library (Dezsõ et al., 2011). For solving all LPs and MIPs we used the commercial solver CPLEX 12.7. All

Instance	$\lvert P\rvert$	$\lvert S\rvert$	$\sum_{s\in S}\lvert N^s\rvert$	$\lvert L\rvert$	$\lvert K\rvert$	$G(V,A), h=8$ $\lvert V\rvert$	$\lvert A^S\rvert$	$\lvert A^T\rvert$	$G(V,A), h=12$ $\lvert V\rvert$	$\lvert A^S\rvert$	$\lvert A^T\rvert$
Baltic	12	3	6	14	22	294	1008	7056	196	518	3136
WAF	20	11	41	43	37	903	7455	62622	602	3570	27832
Mediterranean	39	6	19	53	365	1113	3402	36162	742	1680	16072
Pacific	45	20	97	135	722	2835	20265	313992	1890	9520	139552

Table 4.2: Data instances. Unscheduled liner shipping networks of Karsten et al. (2017a) based on the LINER-LIB instances.

experiments were performed on a system with a Xeon E5-2680 2.8GHz processor and 16 GB memory. The bunker price and demand rejection penalties were chosen to be equal to those of previous LINER-LIB based studies addressing speed optimization, corresponding to 600 USD and 1000 USD, respectively. Parameter G_{cg}, the column generation acceptance gap, was set to 0.001 in all reported tests.

4.7.1 Data instances

In our computational experiments we use data instances from the publicly available LINER-LIB benchmark suite developed by Brouer et al. (2014). The library contains a set of realistic data instances for liner shipping network design and related problems. It does not, however, specify any networks.

The networks we use for our computational experiments correspond to the best networks found by the matheuristic of Karsten et al. (2017b) and are summarized in Table 4.2. As the names suggest, the instances represent networks in the Baltic sea, the Mediterranean sea, along the coast of West-Africa and in the Pacific ocean. Table 4.2 further reports the number of ports ($|P|$), services ($|S|$), vessels ($\sum_{s\in S}|N^s|$), port calls or sailing legs ($|L|$) and demands ($|K|$) for each network. The right section of the table describes the size of graphs $G(V,A)$ of all instances for a time discretization of $h = 8$ hours and $h = 12$ hours. We can see from the reported figures that the number of transshipment arcs exceeds the number of sailing arcs by factors of 5 to 14. The large number of transshipment arcs reflects the additional complexity of not just optimizing speed but determining a schedule for all services.

For all experiments reported below we imposed time limits on the algorithm runtime, loosely correlated with the number of demands per instance. The time limits of instances `Baltic`, `WAF`, `Mediterranean` and `Pacific` were set to 900, 1800, 7200 and 14400 seconds, respectively.

4.7.2 Algorithm performance

The branch-and-price algorithm is fully deterministic, including the integrated primal heuristic. During preliminary tests we observed, however, that the quality of the best upper bound can vary under different parameter choices, particularly for larger instances. In Table 4.3 we report run statistics for all instances obtained from different parameter settings. The results are based on graph $G(V, A)$ based on a time discretization of $h = 8$ hours. Each row corresponds to a particular combination of parameters $I_{\text{cg}}^{\max}$ and $N_{\text{cg}}^{\max}$, i.e. the maximum number of column generation iterations at each node and the maximum number of cargo paths to generate for each demand per iteration. We further report the number of nodes solved and pruned by optimality and bound (no node was pruned by infeasibility). The following three columns contain the global lower bound (LB), the global upper bound (UB) and the optimality gap. The rightmost columns report the total running times (in seconds) spent on the cargo path pricing, on solving the RMP and on running the primal heuristic.

We conducted the same computational experiments using graph $G(V, A)$ with a time discretization of $h = 12$ hours. The resulting models are smaller and thus easier to solve. For instance `Mediterranean`, however, no feasible schedule exists for two out of six services. In Section 4.B we report the results for the other three networks. All results reported in the remainder of this section are based on graph $G(V, A)$ with a time discretization of $h = 8$ hours.

4.7.2.1 Quality of found solutions

As the reported gaps indicate, the algorithm performance varies significantly between instances `Baltic` and `WAF` on one side and `Mediterranean` and `Pacific` on the other side. For the first two networks the best found solutions are close to optimal, whereas large gaps remain for the other two networks. It turns out that the transit time restrictions have a large influence on the gap. In Table 4.4 we compare average and best LB, UB and gap of the runs reported in Table 4.3 (limited transit time) with the corresponding values obtained by running the exact same tests but without any transit time restrictions (unlimited transit time). First, we observe that the best and average upper bounds do almost not differ in case the LSSCAP is solved without any transit time restrictions, i.e. the found upper bounds are consistent over different parameter settings. Second, we note that the gaps are also much smaller. The two observations suggest (not surprisingly) that the problem becomes much easier without transit time restrictions.

Tight transit time limits imply a larger objective function value, because revenue is lost in case some demands cannot be served due to the restrictions. The lower bound of the LP relaxation of the same model, however, is less affected by tighter transit time restrictions, because different fractions of services can operate different schedules and therefore allow a larger number of demands to be served. By comparing the upper bounds under unlimited transit times to those obtained under limited transit times, we can conclude that the transit time limits are rather loose and almost not prohibitive for instances `Baltic` and `WAF`.

For the analysis performed in Sections 4.7.3 and 4.7.4 we will use the best solutions among the ones reported in Table 4.3.

Parameters		Nodes			LB	UB	Gap %	Time		
I_{cg}^{max}	N_{cg}^{max}	solved	pruned					Pricing	RMP	Heur.
			opt.	bound						
Baltic										
3	1	1343	44	317	$-6.46 \cdot 10^5$	$-6.46 \cdot 10^5$	0.1	2.4	17.3	879.1
3	3	1301	38	284	$-6.46 \cdot 10^5$	$-6.46 \cdot 10^5$	0.1	2.1	18.0	878.2
3	5	1317	28	310	$-6.46 \cdot 10^5$	$-6.46 \cdot 10^5$	0.1	2.1	19.1	877.2
3	10	1383	28	372	$-6.46 \cdot 10^5$	$-6.46 \cdot 10^5$	0.1	2.0	23.1	874.0
5	1	1345	39	323	$-6.46 \cdot 10^5$	$-6.46 \cdot 10^5$	0.1	2.5	18.7	876.6
5	3	1328	32	317	$-6.46 \cdot 10^5$	$-6.46 \cdot 10^5$	0.1	2.1	21.1	874.8
5	5	1367	34	351	$-6.46 \cdot 10^5$	$-6.46 \cdot 10^5$	0.1	2.2	21.5	875.0
5	10	1351	32	336	$-6.46 \cdot 10^5$	$-6.46 \cdot 10^5$	0.1	2.1	22.1	873.9
WAF										
3	1	403	0	9	$-8.90 \cdot 10^6$	$-8.74 \cdot 10^6$	1.8	48.1	117.8	1637.6
3	3	450	0	54	$-8.76 \cdot 10^6$	$-8.74 \cdot 10^6$	0.3	42.5	276.6	1483.6
3	5	433	0	37	$-8.76 \cdot 10^6$	$-8.74 \cdot 10^6$	0.2	33.1	305.8	1464.0
3	10	431	0	58	$-8.76 \cdot 10^6$	$-8.74 \cdot 10^6$	0.2	31.9	354.0	1415.0
5	1	440	0	58	$-8.77 \cdot 10^6$	$-8.74 \cdot 10^6$	0.4	73.1	196.0	1533.2
5	3	398	0	14	$-8.76 \cdot 10^6$	$-8.74 \cdot 10^6$	0.2	51.4	286.8	1461.0
5	5	391	0	10	$-8.76 \cdot 10^6$	$-8.74 \cdot 10^6$	0.2	38.1	314.6	1445.5
5	10	382	0	10	$-8.76 \cdot 10^6$	$-8.74 \cdot 10^6$	0.2	30.9	294.7	1474.2
Mediterranean										
3	1	857	0	0	$2.18 \cdot 10^5$	$6.37 \cdot 10^5$	65.7	672.8	5979.8	543.6
3	3	837	0	1	$2.16 \cdot 10^5$	$5.88 \cdot 10^5$	63.3	514.4	6193.4	488.4
3	5	700	0	0	$2.12 \cdot 10^5$	$6.21 \cdot 10^5$	65.8	482.6	6112.5	601.6
3	10	659	0	1	$2.18 \cdot 10^5$	$6.00 \cdot 10^5$	63.7	425.4	6333.4	437.9
5	1	853	0	0	$2.13 \cdot 10^5$	$6.37 \cdot 10^5$	66.5	842.8	5774.7	577.9
5	3	940	0	0	$2.11 \cdot 10^5$	$5.69 \cdot 10^5$	62.8	534.0	6172.9	488.9
5	5	725	0	1	$2.22 \cdot 10^5$	$5.92 \cdot 10^5$	62.5	572.5	6205.2	418.3
5	10	719	0	0	$2.16 \cdot 10^5$	$6.57 \cdot 10^5$	67.2	448.1	5967.8	780.4
Pacific										
3	1	64	0	0	$-1.44 \cdot 10^7$	$-4.52 \cdot 10^6$	218.0	3103.8	11217.0	78.7
3	3	40	0	0	$-1.27 \cdot 10^7$	$-4.75 \cdot 10^6$	167.6	1959.1	12384.8	55.6
3	5	52	0	0	$-1.11 \cdot 10^7$	$-5.12 \cdot 10^6$	117.1	1626.3	12705.4	67.8
3	10	39	0	0	$-1.11 \cdot 10^7$	$-4.27 \cdot 10^6$	160.0	1188.1	13173.2	38.2
5	1	53	0	0	$-1.11 \cdot 10^7$	$-4.39 \cdot 10^6$	153.0	3528.4	10779.9	91.1
5	3	36	0	0	$-1.11 \cdot 10^7$	$-3.52 \cdot 10^6$	216.0	2271.5	12079.1	49.0
5	5	35	0	0	$-1.11 \cdot 10^7$	$-3.22 \cdot 10^6$	245.0	1886.3	12423.6	89.6
5	10	28	0	0	$-1.11 \cdot 10^7$	$-5.50 \cdot 10^6$	101.8	1292.5	13076.5	30.4

Table 4.3: Branch-and-price algorithm results for all instances and different parameter settings. A time discretization of $= 8$ hours was used.

Instance	Global lower bound		Global upper bound		Gap %	
	Unlimited transit time	Limited transit time	Unlimited transit time	Limited transit time	Unlimited transit time	Limited transit time
Baltic						
Best	$-6.46 \cdot 10^5$	$-6.46 \cdot 10^5$	$-6.46 \cdot 10^5$	$-6.46 \cdot 10^5$	0.1	0.1
Avg.	$-6.46 \cdot 10^5$	$-6.46 \cdot 10^5$	$-6.46 \cdot 10^5$	$-6.46 \cdot 10^5$	0.1	0.1
WAF						
Best	$-8.76 \cdot 10^6$	$-8.76 \cdot 10^6$	$-8.75 \cdot 10^6$	$-8.74 \cdot 10^6$	0.1	0.2
Avg.	$-8.79 \cdot 10^6$	$-8.76 \cdot 10^6$	$-8.75 \cdot 10^6$	$-8.74 \cdot 10^6$	0.4	0.2
Mediterranean						
Best	$-2.21 \cdot 10^5$	$2.22 \cdot 10^5$	$-2.18 \cdot 10^5$	$5.69 \cdot 10^5$	1.9	62.5
Avg.	$-2.22 \cdot 10^5$	$2.16 \cdot 10^5$	$-2.17 \cdot 10^5$	$6.09 \cdot 10^5$	2.4	64.5
Pacific						
Best	$-1.31 \cdot 10^7$	$-1.11 \cdot 10^7$	$-1.22 \cdot 10^7$	$-0.55 \cdot 10^7$	6.9	101.8
Avg.	$-1.65 \cdot 10^7$	$-1.13 \cdot 10^7$	$-1.22 \cdot 10^7$	$-0.44 \cdot 10^7$	35.3	165.8

Table 4.4: Effect of transit time restrictions on the solution quality of the branch-and-price algorithm.

4.7.2.2 Runtime analysis

Table 4.3 also reports running times for the most important algorithm components, the pricing problem, the restricted master problem and the primal heuristic. The computational time spend on solving the pricing problems is correlated with the number of demands of an instance. However, even for the largest network `Pacific`, it constitutes only $8.3\% - 24.5\%$ of the total runtime.

The largest amount of time is spent on solving the RMPs and on running the primal heuristic. The split between these two, however, varies significantly between the instances. The heuristic consumes around 80%-98% of the total runtime for the two smaller instances, but only 6%-10% for instance `Mediterranean` and some almost negligible 0.2%-0.6% for instance `Pacific`. Conversely, with $74.9\% - 91.5\%$ most of the runtime of the two larger instances is spent on solving the RMP. Evidently, the primal heuristic scales better to larger instances than the RMP.

The low share of total runtime for the primal heuristic is surprising at first sight. Model (4.21)-(4.25), which is solved as part of the primal heuristic, does not possess the integrality property, but the linear programming solution often turns out to be integral in our experiments. The average share of integral LP solutions varies significantly between instances and lies in the range of $10\% - 98\%$ for selected runs. However, the MIP solver was generally able to identify the optimal integer solution at the root node after adding violated cuts.

Fuel cons. function	Cargo transported	Cargo transshipped	Avg. capacity util.	Revenue	Fuel cost	Handling cost	Rejection penalty	Obj. value
Baltic								
Payload	4312	0	0.795	$36.14{\cdot}10^5$	$3.55{\cdot}10^5$	$20.21{\cdot}10^5$	$5.92{\cdot}10^5$	$-6.46{\cdot}10^5$
Simple	4312	0	0.796	$36.14{\cdot}10^5$	$3.44{\cdot}10^5$	$20.21{\cdot}10^5$	$5.92{\cdot}10^5$	$-6.57{\cdot}10^5$
WAF								
Payload	8342	2134	0.542	$14.69{\cdot}10^6$	$1.96{\cdot}10^6$	$3.79{\cdot}10^6$	$0.20{\cdot}10^6$	$-8.74{\cdot}10^6$
Simple	8342	2134	0.539	$14.69{\cdot}10^6$	$2.14{\cdot}10^6$	$3.78{\cdot}10^6$	$0.20{\cdot}10^6$	$-8.57{\cdot}10^6$
Mediterranean								
Payload	6062	2002	0.564	$4.32{\cdot}10^6$	$1.06{\cdot}10^6$	$2.35{\cdot}10^6$	$1.48{\cdot}10^6$	$5.69{\cdot}10^6$
Simple	6083	2055	0.578	$4.33{\cdot}10^6$	$1.11{\cdot}10^6$	$2.36{\cdot}10^6$	$1.46{\cdot}10^6$	$5.96{\cdot}10^6$
Pacific								
Payload	37593	12525	0.698	$4.03{\cdot}10^7$	$1.23{\cdot}10^7$	$1.59{\cdot}10^7$	$0.66{\cdot}10^7$	$-0.55{\cdot}10^7$
Simple	37448	11230	0.699	$4.01{\cdot}10^7$	$1.29{\cdot}10^7$	$1.58{\cdot}10^7$	$0.67{\cdot}10^7$	$-0.47{\cdot}10^7$

Table 4.5: Solution characteristics under different fuel consumption functions. The first two columns denote the total number of containers (in FEU) transported and transshipped, respectively. Revenues, all costs and the objective value are reported in USD.

4.7.3 Assessment of incorrectly specified fuel consumption functions

To assess the gain from using a more accurate payload dependent fuel consumption function, we additionally solved all LSSCAP instances using the simple, speed-only dependent fuel consumption function (defined by equation (4.1)). Table 4.5 reports different key performance indicators of the four networks for both fuel consumption functions. Recall that the simple fuel consumption function is based on an assumed average load factor of 0.7. Not surprisingly, we observe that for instances with a capacity utilization significantly below 0.7, the fuel cost is lower if the payload dependent fuel consumption function was used, and vice versa. We will analyze the cases `Baltic` and `WAF` in more detail, because the found solutions are provably close to the optimal schedule (all gaps $< 0.2\%$) and thus allow for inference about the impact of the underlying fuel consumption function.

For the smallest instance `Baltic`, the optimal schedules are identical, independent of the fuel consumption function used. The calculated fuel cost, however, is 3.2% lower in case of the speed-only dependent fuel consumption function. The amount of containers transported is the same in both cases, but a single demand of 7 containers is routed differently through the small network of only 3 services. The different routing explains the slight variation in capacity utilization between the two fuel consumption scenarios as reported in Table 4.5.

The solutions for the two considered fuel consumption functions differ more significantly in case of instance `WAF`. The calculated fuel cost in case of the speed-only dependent fuel consumption scenario is 9.1% higher than for the payload dependent fuel consumption scenario. If the model was optimized assuming the simple fuel consumption function, but the fuel consumption was

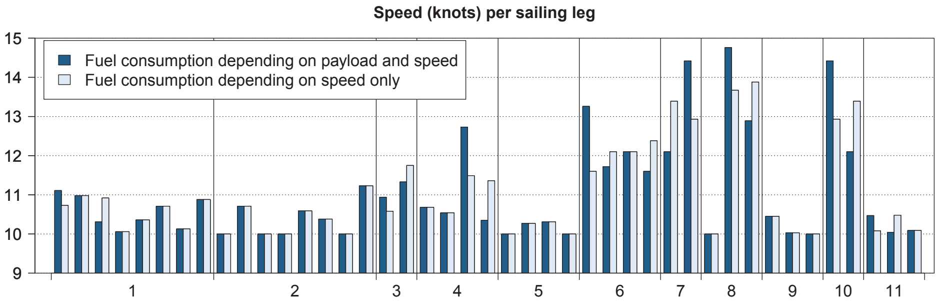


Figure 4.6: Sailing speed per leg in the best found schedules for instance WAF. The bar plot is divided by services and the x-axis labels denote the service. On the majority of services speeds vary per sailing leg between the two specified fuel consumption functions.

calculated post hoc based on the more accurate payload dependent function, the fuel cost would still exceed the fuel cost of the optimal schedule by 1.7%.

Whereas it is quite intuitive that the fuel cost will vary under the two considered fuel consumption functions, it is less trivial that the incorrect specification of the fuel consumption function may result in different, suboptimal service schedules. Figure 4.6 visualizes the optimal speed on each single sailing leg of all `WAF` services for the two fuel consumption specifications. For 8 out of 11 services the optimal speed profiles are different and, not observable in Figure 4.6, the actual berth times are also different for the three services whose speed profiles are equal under both fuel consumption functions. Additionally, multiple cargo routes differ between the two cases, including transshipments at different ports. Again, the different cargo routes explain the variation in capacity utilization. The transshipments in different ports are reflected in a small difference in total cargo handling costs.

The instances `Baltic` and `WAF` are relatively small, particularly in the number of demands. Still, the results indicate that an incorrectly specified fuel consumption function can lead to suboptimal schedule design and cargo routing decisions.

4.7.4 Exact schedules vs. approximated transshipment times

A significant number of studies that address operational or tactical problems faced by liner shipping companies make the assumption that the time needed for transshipping containers between two services is constant. Commonly this assumption is made because the integration of scheduling decisions makes many problems much harder both in terms of complexity as well as algorithmic tractability. The assumption of constant transshipment times per se may not represent a flaw, if it does not imply misleading conclusions. Intuitively, if transit time limits apply to a particular demand, increasing the constant transshipment time in a model reduces

Instance	Transshipment time	Cargo transp.	Cargo transship.	Avg. cap. util.	Revenue	Fuel cost	Handling cost	Rejection penalty	Obj. value
WAF	exact	8342	2134	54.18	$14.69 \cdot 10^6$	$1.96 \cdot 10^6$	$3.79 \cdot 10^6$	$0.20 \cdot 10^6$	$-8.74 \cdot 10^6$
	48	8342	2134	54.20	$14.69 \cdot 10^6$	$1.96 \cdot 10^6$	$3.78 \cdot 10^6$	$0.20 \cdot 10^6$	$-8.75 \cdot 10^6$
	72	8342	2134	54.18	$14.69 \cdot 10^6$	$1.98 \cdot 10^6$	$3.78 \cdot 10^6$	$0.20 \cdot 10^6$	$-8.73 \cdot 10^6$
	96	8337	2129	54.12	$14.68 \cdot 10^6$	$2.01 \cdot 10^6$	$3.78 \cdot 10^6$	$0.20 \cdot 10^6$	$-8.69 \cdot 10^6$
	120	7805	1597	49.98	$13.74 \cdot 10^6$	$1.98 \cdot 10^6$	$3.50 \cdot 10^6$	$0.74 \cdot 10^6$	$-7.53 \cdot 10^6$
Med	exact	6062	2002	56.41	$4.32 \cdot 10^6$	$1.06 \cdot 10^6$	$2.35 \cdot 10^6$	$1.48 \cdot 10^6$	$0.57 \cdot 10^6$
	48	6383	2512	61.52	$4.53 \cdot 10^6$	$1.06 \cdot 10^6$	$2.56 \cdot 10^6$	$1.16 \cdot 10^6$	$0.25 \cdot 10^6$
	72	6324	2371	60.39	$4.50 \cdot 10^6$	$1.07 \cdot 10^6$	$2.51 \cdot 10^6$	$1.22 \cdot 10^6$	$0.30 \cdot 10^6$
	96	6106	2083	56.73	$4.28 \cdot 10^6$	$1.06 \cdot 10^6$	$2.37 \cdot 10^6$	$1.44 \cdot 10^6$	$0.59 \cdot 10^6$
	120	5903	1824	53.92	$4.19 \cdot 10^6$	$1.05 \cdot 10^6$	$2.27 \cdot 10^6$	$1.64 \cdot 10^6$	$0.77 \cdot 10^6$
Pacific	exact	37593	12525	69.77	$4.03 \cdot 10^7$	$1.23 \cdot 10^7$	$1.59 \cdot 10^7$	$0.66 \cdot 10^7$	$-0.55 \cdot 10^7$
	48	39961	15323	73.74	$4.31 \cdot 10^7$	$1.19 \cdot 10^7$	$1.70 \cdot 10^7$	$0.42 \cdot 10^7$	$-1.00 \cdot 10^7$
	72	39573	15974	73.93	$4.28 \cdot 10^7$	$1.26 \cdot 10^7$	$1.69 \cdot 10^7$	$0.46 \cdot 10^7$	$-0.86 \cdot 10^7$
	96	38283	13399	72.23	$4.13 \cdot 10^7$	$1.28 \cdot 10^7$	$1.66 \cdot 10^7$	$0.59 \cdot 10^7$	$-0.61 \cdot 10^7$
	120	36520	10200	68.28	$3.92 \cdot 10^7$	$1.23 \cdot 10^7$	$1.57 \cdot 10^7$	$0.77 \cdot 10^7$	$-0.36 \cdot 10^7$

Table 4.6: Solution characteristics for different transshipment time approximations.

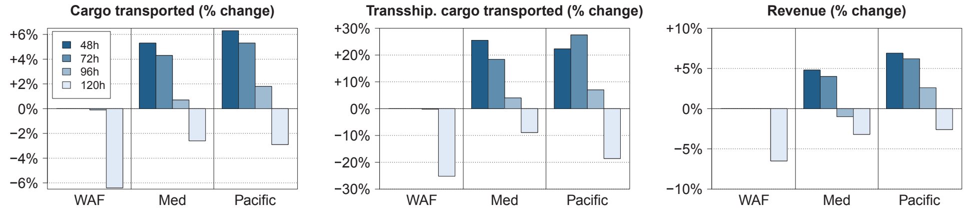
the remaining transportation time in case a cargo path requires one or more transshipments. Conversely, underestimating constant transshipment times may lead to an underestimation of total transit times. In the following, we try to answer the question of what a good approximation for transshipment times is, if it cannot be modeled exactly. Equivalently, we analyze the impact a wrong choice may have.

To simulate a model of constant transshipment times, we fixed the duration of all transshipment arcs $a \in A^T$ to a constant value. We solved the LSSCAP for constant transshipment times of 48, 72, 96 and 120 hours. It is noteworthy that the LSSCAP becomes significantly easier under the constant transshipment time assumption, as it allows to fix one port call on each service.

Table 4.6 reports, for each instance, key figures of the solutions obtained under different transshipment time assumptions. Note that we do not include instance `Baltic` in the analysis, because in all solutions all cargo is shipped directly, i.e. without any transshipment, and variations in transshipment times therefore do neither affect the scheduling nor the cargo routing.

Figure 4.7 visualizes the influence of different constant transshipment time values on selected key performance indicators. The figures show the relative change of selected indicators compared to the solutions of the base model with exact transshipment times. Recall that the original model assumes a *minimum* transshipment time of 48 hours, i.e. the simultaneous exchange of cargo between two vessels is not feasible.

Not surprisingly, different constant transshipment time values primarily affect the transshipped cargo, and changes in total cargo transported (Figure 4.7a) result almost exceptionally from



(a) Number of containers transported (b) Number of transshipped containers transported (c) Revenue

Figure 4.7: Comparison of different constant transshipment times and how they influence solutions to the LSSCAP.

changes in transshipped cargo transported (Figure 4.7b).

4.8 Concluding remarks

By combining the interdependent problems of liner service scheduling and cargo allocation we considered a previously unstudied planning problem for liner shipping companies. To solve the problem, we developed a model and an exact branch-and-price algorithm with an integrated primal heuristic. The model further stands out for incorporating a more realistic payload dependent fuel consumption function.

Based on the computational results we can answer both research questions asked at the beginning of Section 4.7 in the affirmative: First, an incorrectly specified fuel consumption function that neglects payload *can* result in the design of suboptimal schedules, besides providing wrong cost estimates. The extent to which an incorrect fuel consumption affects the quality of solutions to scheduling or speed optimization problems, however, will depend on various factors as e.g. the variance of payloads or the service level requirements. Second, not explicitly considering service schedules but approximating transshipment times can lead to heavily under- or overestimated transit times of transshipped cargo. In our computational tests, with demand and thus revenue being perfectly elastic with regard to transit time, different transshipment time approximations result in substantially different evaluations of scheduled liner shipping networks.

The computational experiments and analysis of results have further shown that the LSSCAP is a difficult combinatorial problem, particularly if the transit time limits of a large number of demands are very tight.

Based on our findings we can suggest different directions for future research. The computational experiments are based on the LINER-LIB data instances that treat transit time limits for origin-destination paths as hard and exogenously given limits. In practice, transit times and freight rates are commonly contractually agreed upon. A more accurate way of accounting for the time sensitivity of demand is to internalize the shippers inventory cost, as e.g. suggested by Psaraftis and Kontovas (2014). Further empirical research, however, is necessary to obtain reasonable estimates of the form and parameters of the revenue function of transit time.

The presented method performs very well for the less constrained instances, but the remaining gaps for the more constrained instances are very large. Although we conjecture that the found solutions are of good quality, further work is necessary to prove it. Particularly solving the LP relaxation of the restricted master problem at each column generation iteration consumes a significant amount of running time for the more difficult instances, but the resulting lower bounds are still very weak. The combination of column generation and Lagrangian relaxation has successfully been used to speed up column generation for other applications (see e.g. Huisman et al., 2005) and represents a worthy research avenue for the problem at hand.

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4.A Estimation of measurements of LINER-LIB vessel classes

In order to derive a speed and payload dependent fuel consumption function of type (4.2) for each LINER-LIB vessel class, we first need estimates of each vessel class' lightweight lwt_e and average weight owt_e of fuel, ballast water, fresh water and crew. None is specified by the LINER-LIB.

Kristensen (2013) and Kristensen (2012) derive estimation functions for all main dimensions of container vessels that are based on a vessel's capacity in TEU. The estimators were obtained through regression analysis based on IHS Fairplay data. Below we state all estimation functions from Kristensen (2013) and Kristensen (2012) that we used to derive the required vessel class measurements for the LINER-LIB vessel classes. Finally, all resulting estimates for the LINER-LIB vessel classes are presented in Table 4.7.

Length between perpendiculars (m)

$$lpp(cap) = \begin{cases} 10.14 \cdot cap^{0.378} & \text{if Feeder} \\ 2.494 \cdot cap^{0.555} & \text{if Panamax } (cap \leq 3800) \\ 241.92 + 0.037 \cdot (cap - 3800) & \text{if Panamax } (cap > 3800) \\ 131.31 + 0.03012 \cdot cap - 9.9556 \cdot 10^{-7} \cdot cap^2 & \text{if Post Panamax } (cap \leq 8000) \\ 14.66 \cdot cap^{0.339} & \text{if Post Panamax } (cap > 8000) \end{cases} \tag{4.28}$$

Breadth (m)

$$breadth(cap) = \begin{cases} 2.9 \cdot cap^{0.3} & \text{if Feeder} \\ 32.22 & \text{if Panamax} \\ 32.51 + 0.0013 \cdot cap & \text{if Post Panamax} \end{cases} \tag{4.29}$$

Depth (m)

$$depth(cap) = \begin{cases} 0.767 \cdot cap^{0.394} & \text{if Feeder} \\ 14.77 + 0.0013 \cdot cap & \text{if Panamax} \\ \min\{30.2; 16.5 + 0.0011 \cdot cap\} & \text{if Post Panamax} \end{cases} \tag{4.30}$$

Lightweight coefficient

$$\beta^{\text{lwt}}(cap) = \begin{cases} 0.659 \cdot cap^{-0.23} & \text{if Feeder} \\ 0.105 & \text{if Panamax} \\ \max\{0.09; 0.104 - 1.15 \cdot 10^{-6} \cdot cap\} & \text{if Post Panamax} \end{cases} \tag{4.31}$$

Vessel class	cap_e	lpp_e	$breadth_e$	$depth_e$	β_e^{lwt}	lwt_e	dwt_e	pl_e	owt_e
Feeder_450	900	132.7	22.3	11.2	0.14	4567	12285	0.66	4235
Feeder_800	1600	164.9	26.5	14.0	0.12	7413	21840	0.67	7268
Panamax_1200	2400	187.5	32.2	17.9	0.11	11346	32851	0.68	10486
Panamax_2400	4800	278.9	32.2	21.0	0.11	19825	61325	0.72	17073
Post_Panamax	8400	313.7	43.4	25.7	0.09	33079	99789	0.78	21674
Super_Panamax	15000	381.8	52.0	30.2	0.09	53972	168000	0.90	17640

Table 4.7: Estimated main dimensions of LINER-LIB vessel classes based on estimators by Kristensen (2012) and Kristensen (2013).

Lightweight (t)

$$lwt(cap) = \beta^{\text{lwt}}(cap) \cdot lpp(cap) \cdot breadth(cap) \cdot depth(cap) \tag{4.32}$$

Maximum permissible deadweight (t)

$$dwt(cap) = \begin{cases} 13.56 \cdot cap & \text{if Feeder} \\ 14.6 \cdot cap - 0.00038 \cdot cap^2 & \text{if Panamax} \\ \max\{11.2 \cdot cap; 50.43 \cdot cap^{0.84}\} & \text{if Post Panamax} \end{cases} \tag{4.33}$$

Maximum payload as factor of deadweight

$$pl(cap) = 0.64 + 0.000017 \cdot cap \tag{4.34}$$

Weight of fuel, ballast water, fresh water, provisions and crew (t)

$$owt(cap) := (1 - pl(cap)) \cdot dwt(cap) \tag{4.35}$$

4.B Computational results for a time discretization of 12 hours

Table 4.8 reports computational results based on graph $G(V, A)$ with a time discretization of $h = 12$. The choice of time discretization may affect the solution quality because a different time discretization implies a different set of feasible speeds between ports. Generally, a coarser time discretization can be expected to result in worse *optimal* solutions as it implies a smaller set of feasible speeds between ports. The optimal solution obtained for a coarser time discretization is always worse or at best equal to the optimal solution for a finer time discretization, if the coarser time discretization is a multiple of the finer one. $h = 12$ is not a multiple of $h = 8$ and as we observe for instance `Baltic`, it may lead to better results. The best found upper bounds for instance `Pacific` are, on average, much better under $h = 12$ than under $h = 8$.

Parameters		Nodes			LB	UB	Gap %	Time		
I_{cg}^{max}	N_{cg}^{max}	solved	pruned					Pricing	RMP	Heur.
			opt.	bound						
Baltic										
3	1	765	0	277	$-6.49{\cdot}10^5$	$-6.49{\cdot}10^5$	0.0	0.1	2.3	116.2
3	3	767	0	284	$-6.49{\cdot}10^5$	$-6.49{\cdot}10^5$	0.0	0.1	2.2	116.2
3	5	793	0	286	$-6.49{\cdot}10^5$	$-6.49{\cdot}10^5$	0.0	0.1	2.5	121.2
3	10	783	0	283	$-6.49{\cdot}10^5$	$-6.49{\cdot}10^5$	0.0	0.1	2.5	118.5
5	1	747	0	274	$-6.49{\cdot}10^5$	$-6.49{\cdot}10^5$	0.0	0.1	2.2	110.3
5	3	753	0	277	$-6.49{\cdot}10^5$	$-6.49{\cdot}10^5$	0.0	0.1	2.3	112.2
5	5	797	0	293	$-6.49{\cdot}10^5$	$-6.49{\cdot}10^5$	0.0	0.1	2.5	122.6
5	10	741	1	274	$-6.49{\cdot}10^5$	$-6.49{\cdot}10^5$	0.0	0.1	2.4	111.6
WAF										
3	1	738	350	0	$-8.74{\cdot}10^6$	$-8.73{\cdot}10^6$	0.2	15.3	64.6	1717.9
3	3	722	352	0	$-8.74{\cdot}10^6$	$-8.73{\cdot}10^6$	0.2	11.4	83.8	1705.6
3	5	733	349	0	$-8.74{\cdot}10^6$	$-8.73{\cdot}10^6$	0.2	11.0	74.0	1715.1
3	10	738	358	0	$-8.74{\cdot}10^6$	$-8.73{\cdot}10^6$	0.1	11.1	83.3	1704.4
5	1	714	352	0	$-8.74{\cdot}10^6$	$-8.73{\cdot}10^6$	0.2	16.0	74.0	1708.7
5	3	710	350	0	$-8.74{\cdot}10^6$	$-8.73{\cdot}10^6$	0.1	11.4	107.9	1682.1
5	5	712	355	0	$-8.74{\cdot}10^6$	$-8.72{\cdot}10^6$	0.2	9.6	40.2	1749.7
5	10	710	352	0	$-8.74{\cdot}10^6$	$-8.73{\cdot}10^6$	0.2	10.2	60.1	1731.1
Pacific										
3	1	212	106	0	$-10.95{\cdot}10^6$	$-5.92{\cdot}10^6$	85.0	1489.7	12716.8	192.4
3	3	203	102	0	$-10.95{\cdot}10^6$	$-5.76{\cdot}10^6$	90.1	1208.8	13053.7	136.4
3	5	187	94	0	$-10.95{\cdot}10^6$	$-5.27{\cdot}10^6$	107.8	1032.4	13261.9	104.8
3	10	173	87	0	$-10.95{\cdot}10^6$	$-4.62{\cdot}10^6$	136.9	789.4	13510.1	99.5
5	1	207	104	0	$-10.95{\cdot}10^6$	$-5.66{\cdot}10^6$	93.5	1860.8	12390.8	147.1
5	3	169	85	0	$-10.95{\cdot}10^6$	$-5.84{\cdot}10^6$	87.4	1317.9	12968.3	112.7
5	5	169	85	0	$-10.95{\cdot}10^6$	$-5.49{\cdot}10^6$	99.3	1159.8	13071.8	167.2
5	10	141	71	0	$-10.96{\cdot}10^6$	$-4.06{\cdot}10^6$	169.7	906.0	13386.6	106.4

Table 4.8: Branch-and-price algorithm results for a time discretization of $h = 12$ hours.

Chapter 5

The Liquefied Natural Gas Infrastructure and Tanker Fleet Sizing Problem

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Abstract

We consider a strategic infrastructure and tanker fleet sizing problem in the liquefied natural gas business. The goal is to minimize long-term on-shore infrastructure and tanker investment cost combined with interrelated expected cost for operating the tanker fleet. A non-linear arc-based model and an exact solution method based on a set-partitioning formulation are developed. The latter approach allows very fast solution times. Computational results for a case study with a liner shipping company are presented, including an extensive sensitivity analysis to account for limited predictability of key parameter values, to analyze the solutions' robustness and to derive basic decision rules.

5.1 Introduction

In 2008 the International Maritime Organization (IMO), a specialized agency of the United Nations, has introduced new regulations for the prevention of pollution from ships that aim at reducing the emission of sulphur oxides, amongst others (IMA, 2008). The regulations already apply in so-called *Emission Control Areas* (ECA) since 2015 and in late 2016 the IMO confirmed that the low-sulphur fuel oil requirements will become binding globally in 2020. Currently the majority of ships (80-85%, Chryssakis et al., 2014), including container vessels, are run on heavy fuel oil (HFO). As a consequence of the new limits on sulphur emissions, many of today's ships will no longer be able to operate as of today, because emissions due to the use of HFO exceed the limits.

In various industrial strategic papers and research studies (see e.g. Andersen et al., 2013; Rozmarynowska and Oldakowski, 2012; Chryssakis et al., 2014) three viable solutions to meet the new requirements have been identified. The first one are exhaust gas aftertreatment systems as e.g. scrubbers. The installation of scrubbers, however, is costly and requires additional space on the ship. Further, they can increase the fuel consumption of a vessel by 2-3% (Chryssakis et al., 2014). The second and most straightforward solution is the use of cleaner marine diesel oil (MDO) or marine gas oil (MGO), as these can usually be used without the need of any modification to the vessels. MDO and MGO are, however, about 1.5-2.0 times more expensive than HFO, with prices expected to increase even further once the sulphur emission limits apply globally. The third solution is the one that motivates our study and considers liquefied natural gas (LNG) as an alternative fuel. LNG is natural gas that is converted to liquid form by cooling it down to approximately $-162°C$ $(-260°F)$. It is the cleanest form of fossil fuels and if used to fuel ships, no further measures are needed to satisfy the new regulations concerning the emission of pollutants. As prices are expected to be lower than those for other low-sulfur fuels in the future, LNG is considered a realistic option for deep sea trades in the long term, particularly for liner trades (Lloyd's Register, 2012). However, it requires newbuild vessels as well as additional LNG infrastructure at the ports.

Liner shipping networks consist of cyclic shipping routes, called *services*, that are operated periodically. Figure 5.1 shows an example liner shipping service that connects Asia and Europe. The individual services are connected through ports, where cargo can be transshipped between different services, and thus provide an extensive, wide-ranging transportation network. Typically each port on a service will be visited once per week and the container liner company publishes the weekly berthing time for each port. As a single round trip can take several weeks, each service is operated by a corresponding number of container vessels. The structure and way of operating container shipping networks is very similar to that of buses in public transport, with the containers being the equivalent of passengers. For an introduction to liner shipping see e.g. Stopford (2009).

The current lack of LNG infrastructure for marine bunkering and the uncertainty about future

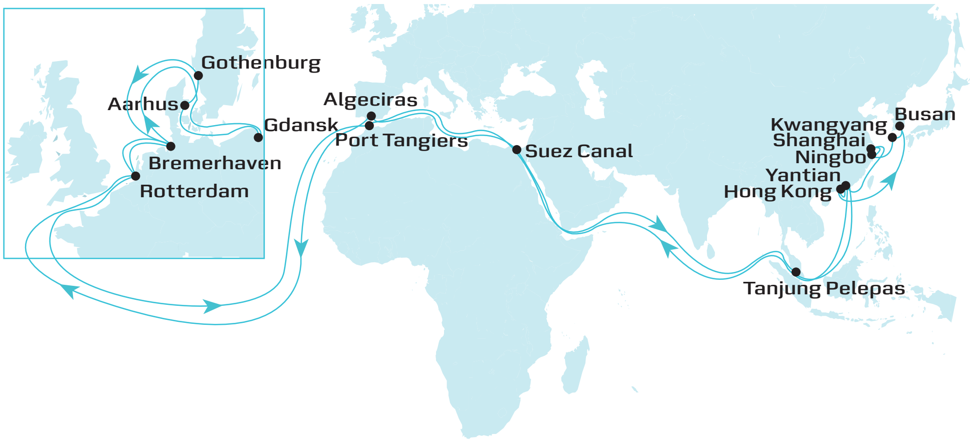


Figure 5.1: Example of a liner shipping service between Asia and Europe. The whole round trip takes 12 weeks and hence 12 vessels are operating on the service to ensure a weekly frequency.

availability is a major drawback of using LNG as a fuel for liner shipping companies. This work is motivated by and based on a case study with a major liner shipping company that considers filling that gap by building up and operating the needed infrastructure by themselves. The study forms the basis of a future scenario in which the company uses LNG fueled container vessels on some of their services.

The liner shipping company is responsible for the transport of LNG to predetermined ports where container vessels will refuel. The transport is done via sea using special purpose LNG tankers. The tanker fleet needs to be ordered or long-term chartered by the company. Furthermore, the infrastructure at the majority of the ports of demand will be built and run by the liner shipping company. The problem is of strategic nature with a time horizon of 2-15 years and combines strategic infrastructure and tanker investment decisions with tactical tanker routing and inventory management decisions. The lack of existing infrastructure allows to simultaneously optimize strategic investment decisions and interdependent tactical decisions.

Strategic infrastructure planning and tactical planning of operations have traditionally been looked at separately in the maritime sector. For tactical and operational problems, the infrastructure and the fleet of vehicles is usually fixed to a large extent. Furthermore, strategic decisions often do not solely depend on quantifiable parameters but are subject to many qualitative arguments (legal issues, local regulations, political decisions, etc). This work aims at providing decision rules of thumb and identifying important relationships between operational/tactical and strategic decisions for the problem studied. The presented models also allow to evaluate manually developed solutions and their sensitivity to changes in input parameters. Hence, an important requirement of our industrial collaborator towards the solution method are fast running times that allow to evaluate large numbers of different scenarios within reasonable time. A second challenge was the requirement towards the solution approach to handle non-linear cost functions that can easily be replaced when better cost estimating functions are obtained during the planning phase.

In a more general context, the problem studied can be considered as a strategic supply chain design problem (see e.g. Vidal and Goetschalckx, 1997, for an overview). Literature on infrastructure planning in the LNG business does hardly exist. The only study the authors are aware of and thus the closest to our work is the one by Jokinen et al. (2015). They present a mixed integer linear programming model that aims at minimizing the cost of a small-scale LNG supply chain in southern Finland, including both annual terminal investment as well as transportation costs. LNG needs to be distributed from a large regasification terminal to several inland consumers, using both sea and land based transport through smaller satellite LNG terminals along the coast and LNG trucks that connect the ports with the points of demand. The application of their proposed model, however, is very limited, as it is tailored towards their particular problem instance with a single supply port and all ports of demand being located along the Finnish coast line west of the supply port. Hence, the routing decision is not part of their model but a simple rule is used. The annual quantities that need to be distributed are rather small and can be met by a single $15\,600\text{m}^3$ LNG tanker. Last but not least, the cost and time distribution of short-sea shipping problems usually are different from deep-sea shipping problems. Even though the focus and scale of their study differ significantly from our work, their results underline the importance of simultaneously considering strategic investment and tactical routing decisions in the still underdeveloped LNG business.

The subproblem of defining the right number and types of LNG tankers to serve the expected demand represents a single-period maritime fleet size and mix problem. Maritime fleet size and mix problems (MFSMPs) represent an own class of problems studied in the literature. We refer the reader to Pantuso et al. (2014) who provide a detailed and recent literature review of papers addressing different types of MFSMPs.

The tactical part of our problem resembles a simple maritime inventory routing problem (MIRP). In fact, if the strategic infrastructure and tanker investment decisions were fixed, the remaining problem would classify as a MIRP. Quite a significant number of contributions to the MIRP literature with applications in the LNG business have been made during recent years. In fact, around 25% of the literature presented in a recent survey on MIRPs by Papageorgiou et al. (2014) are LNG inventory routing problems (LNG-IRP). Grønhaug and Christiansen (2009), Grønhaug et al. (2010) and Andersson et al. (2016) present different exact solution approaches for an LNG-IRP. Exact and heuristic solution approaches for different variations of LNG annual delivery program planning (ADP) problems have been developed by Rakke et al., 2011, Goel et al. (2012), Stålhane et al. (2012), Halvorsen-Weare and Fagerholt (2013), Halvorsen-Weare et al. (2013), Andersson et al. (2015), Goel et al. (2015), Rakke et al., 2015, Shao et al. (2015) and, more recently, Mutlu et al. (2016) and Al-Haidous et al. (2016). The paper by Andersson et al. (2010) provides a general description of the LNG supply chain and presents two related problems.

A more general overview of maritime inventory routing problems together with examples of applications can be found in Christiansen and Fagerholt (2009) or, more recently, Papageorgiou et al. (2014). What these studies have in common is that they address tactical or operational problems with planning horizons of at most 1-2 years. Different to our work, the onshore infrastructure and the available fleet of LNG tankers is generally assumed to be given and fixed. Our proposed strategic models, on the other side, neglect some operational aspects, for example limited berth capacities at ports, delivery time windows, inventory availability at loading ports or stochasticity in short-term demand. The strategic infrastructure and tanker fleet sizing problem and annual delivery planning problems actually complement each other; as suggested by Goel et al. (2012) and Papageorgiou et al. (2015), efficient solution approaches for ADP problems can be used to evaluate strategic infrastructure decisions, given the availability of operational data.

The significant number of recently published industrial studies and surveys that address the feasibility and prospects of LNG as a fuel in deep sea container shipping demonstrate the practical relevance of the problem studied (e.g. DNV-GL, 2014; Andersen et al., 2013; Lloyd's Register, 2012; Lloyd's Register, 2014).

The contribution of this paper is twofold: firstly, we introduce the infrastructure and tanker fleet sizing problem that combines strategic and tactical decisions. We present two mathematical models for solving the problem, with the latter formulation allowing very short solution times. Secondly, we report computational results from an industrial case study. The results show that it pays off to optimize for strategic and tactical decisions simultaneously, because in the long perspective transportation costs can be reduced significantly through optimal investment decisions. The sensitivity analysis indicates that the optimal solution to the base case remains very close to optimal even if input parameters like external cost factors change significantly. On the other hand, the results indicate that increasing cost can hardly be absorbed, even if the tanker fleet or infrastructure capacities could be adjusted over time.

In Section 5.2 the problem is described in detail and corresponding assumptions are introduced. In Section 5.3 we present an arc-based model for conceptual reasons and derive a path based formulation that is used for the computational tests. The case study, its underlying data and cost and corresponding computational results are presented in section 5.4. Concluding remarks and a discussion of the results and potential future work are provided in Section 5.5.

5.2 Problem description

The typical LNG supply chain in a maritime context can be described as follows (see e.g. Andersson et al., 2010): natural gas is extracted, purified and then turned into liquefied natural gas at a designated liquefaction plant. After liquefaction the LNG is stored in full-containment tanks that keep the gas in its liquefied state. It is then transported by special-purpose LNG tankers to its destination terminal, where it is unloaded into onshore storage tanks again. Nowadays most

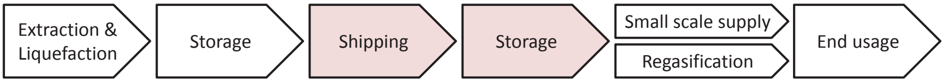


Figure 5.2: The LNG supply chain and the part considered in this work (following Andersson et al., 2010)

Figure 5.3: Example scenario with Qatar as loading port and eight different ports of demand located in Asia and Europe.

of the LNG is regasified at its destination and then distributed to end customers via pipelines or trucks. In our scenario, however, a large portion of the liquefied natural gas would finally be used to fuel container vessels (see Figure 5.2). The container ships would either be refueled at ports with LNG bunkering facilities or by LNG bunker vessels.

All the LNG needed at the ports of demand in this study is considered to be extracted as natural gas (NG) and transformed into LNG at liquefaction plants by third parties. Even though we consider a single supply port in the case study presented in detail in Section 5.4.3, we address the more general case of multiple ports of supply. At the ports of supply the LNG is loaded onto LNG tankers that are either owned or long-term chartered by the liner shipping company. The destinations of the LNG are ports at which LNG fueled container vessels berth and get refueled. The number of ports that shall provide LNG refueling services and their locations are input to the model and in practice are based on estimates of how large the LNG tanks on the container vessels will be and how often the ships will have to refuel. Generally, it is expected that LNG fueled container vessels refuel more often because of the lower energy content per unit of volume. Depending on the LNG tank type used, the compartment housing of the tanks can be up to four times that of HFO tanks of equivalent energy content (Hannula et al., 2006). Figure 5.3 illustrates an example of eight unloading ports and one loading port, as used in the case study presented in Section 5.4.

The annual amount of LNG bought at the supply ports equals the estimated annual demand at all unloading ports. The demand estimates cover the demand for LNG as fuel for container vessels, but may also include some spot market demand at the ports. The responsibility for the transport of the LNG from the loading ports to the unloading ports lies at the operator of the liner ships and is thus part of the decision problem. In accordance with the fixed frequency of liner shipping services and hence constant LNG consumption rates over time, a constant rate of demand for LNG is assumed at the ports over the year. On an operational level fluctuations in demand may occur, for example, because of varying load factors (e.g. before/during Chinese New Year), unplanned deviations from the route due to bad weather or delayed port operations that require the vessels to speed up. Different ways of counteracting temporal excess demand or excess supply exist in practice; LNG tankers can, for example, operate at higher or lower speeds to adjust the rate of supply; another option is to balance excess demand or supply through buying or selling LNG on the steadily growing spot market. Given the assumption of a stable demand for LNG in the medium term, we will assume periodic shipping routes for the LNG tankers as well.

The problem contains two strategic decision problems: first, the choice of the fleet of LNG tankers and, second, the determination of the onshore infrastructure capacity at the receiving ports. A simplified tactical maritime inventory routing problem is solved simultaneously in order to optimize expected operational costs and strategic costs simultaneously.

The first strategic decision problem represents a single-period maritime fleet size and mix problem and consists in selecting the right number and types of LNG tankers from a discrete set of tanker types. The number of available tankers per type can be limited (e.g. if the company already owns tankers or the tankers available on the market at the time of purchase are limited) or assumed to be unlimited (e.g. if the company plans to order newly built LNG tankers). The LNG tankers can either be owned or long-term chartered by the company, in both cases we assume a charter rate per tanker type.

The second strategic decision addresses the infrastructure and its capacity that has to be built up at the ports of demand. The larger the LNG capacity of the infrastructure at a port, the larger are the amounts of LNG that can be unloaded and the less often the depot has to be served by a tanker, given a constant consumption rate.

The maritime inventory routing problem that is part of the overall problem can be classified as a single product MIRP with multiple loading ports, multiple unloading ports and constant production and consumption rates. Due to the strategic nature of the problem considered, we neglect the loss of LNG because of boil-off. The routes that are constructed by the model are not necessarily going to be used in an operational setting; they are constructed in order to estimate transportation cost. In an operational setting the routes would be recomputed taking into account a larger set of constraints, but also allowing more flexibility, for example, with respect to the periodicity of the tanker routes.

We are primarily interested in the interplay of expected operational cost and investment cost under different scenarios. As a summary, the key decisions to take consist in choosing the right onshore infrastructure capacities at the receiving ports and the right tanker fleet to deliver the LNG, both in terms of capacities. Simultaneously, a routing problem is solved such that the routes both satisfy the annual demand and are compatible with the chosen infrastructure options. The goal is to determine the requirements towards the analyzed LNG supply chain while minimizing the combined annual investment cost and operational cost in the long and medium term.

5.3 Modelling and solution method

We first present a non-linear arc-based formulation of a simplification of the underlying problem (sec. 5.3.1). We do not solve the arc-based model, but use it to conceptually introduce the problem to the reader. A very similar model has been tested by Boleda Molas (2015) but turned out to be hard to be solved in reasonable time. In Section 5.3.2 we derive a path-based model that allows for a higher level of details incorporated and also has a much simpler structure. The path based formulation relies on the prior generation of partial solutions, which will be described in Section 5.3.3.

5.3.1 Arc-based model formulation

Let $\mathcal{P}^{\mathrm{D}}$ and $\mathcal{P}^{\mathrm{S}}$ denote the set of ports of demand and the set of ports of supply, respectively. The set of all ports is denoted by $\mathcal{P} = \mathcal{P}^{\mathrm{D}} \cup \mathcal{P}^{\mathrm{S}}$. Let $\mathcal{G}(\mathcal{P}, \mathcal{A})$ denote the corresponding graph with the node set $\mathcal{P}$ and the set of directed arcs $\mathcal{A} = \{(i,j) : i \in \mathcal{P}, j \in \mathcal{P}^{\mathrm{D}}\} \cup \{(i,j) : i \in \mathcal{P}^{\mathrm{D}}, j \in \mathcal{P}\}$, representing sailing legs between all pairs of ports excluding arcs between supply ports. The set of available LNG tanker types is given by $\mathcal{M}$ and indexed by m. Each tanker type is uniquely defined by its capacity q_m and a maximum number of tankers available $\bar{n}_m$ (if applicable). Since we assume that the demand of each port $i \in \mathcal{P}^{\mathrm{D}}$ is fulfilled by exactly one tanker route, the output of the model will be at most $|\mathcal{P}^{\mathrm{D}}|$ routes. We let $\mathcal{R} = \{1, \ldots, |\mathcal{P}^{\mathrm{D}}|\}$ be the index set of the routes resulting from the model.

The annual demand for LNG at each port of demand $i \in \mathcal{P}^{\mathrm{D}}$ is denoted by parameter d_i. We assume a constant sailing speed for each tanker type and hence can derive a time parameter t_{ijm} for each sailing leg $(i,j) \in \mathcal{A}$ and tanker type $m \in \mathcal{M}$. The time includes the sailing time between ports $i \in \mathcal{P}$ and $j \in \mathcal{P}$ but also pilot-out and pilot-in times at the origin and destination port and the time for loading LNG (if $i \in \mathcal{P}^{\mathrm{S}}$) or unloading LNG (if $i \in \mathcal{P}^{\mathrm{D}}$) at the origin port i. The parameter B denotes a relative buffer capacity of LNG storage tanks.

Different cost factors are considered in the model and represented by a potentially non-linear function. This cost function can incorporate any relevant cost factor, as for example annual tanker investment costs or charter rates, annual infrastructure investment and running costs, and tanker routing costs as e.g. tanker fuel costs and port call and canal fees. The annual charter costs generally depend on the number and types of tankers used and possibly, but not necessarily, on the tankers' utilization. The annual cost for building and running the onshore infrastructure generally depends on its capacity and may vary between different ports. The fuel costs will depend on the particular sailing legs and used tanker types as well as the time spent in ports (for in-port fuel consumption). Port call costs and canal transit fees depend on the type of tankers and their actual routings.

The binary decision variables x_{ijrm} denote whether tanker type $m \in \mathcal{M}$ is used on sailing leg $(i, j) \in \mathcal{A}$ in route $r \in \mathcal{R}$ or not. The frequency of each route, i.e. how often the route is operated per year, is represented by the continuous decision variable f_r. The continuous variable y_i denotes the infrastructure capacity at port $i \in \mathcal{P}^{\mathrm{D}}$, on which the infrastructure cost at a port depends on. The number of tankers of type $m \in \mathcal{M}$ needed on route $r \in \mathcal{R}$ is given by the integer decision variable n_{rm}.

The problem can then be formulated as follows:

$$\min c(f_r, n_{rm}, x_{ijrm}, y_i) \tag{5.1}$$

$$\text{s.t.} \quad \sum_{j\in\mathcal{P}}\sum_{r\in\mathcal{R}}\sum_{m\in\mathcal{M}} x_{ijrm} = 1 \qquad \forall i \in \mathcal{P}^{\mathrm{D}} \tag{5.2}$$

$$\sum_{j\in\mathcal{P}} x_{jirm} - \sum_{j\in\mathcal{P}} x_{ijrm} = 0 \qquad \forall i \in \mathcal{P}, r \in \mathcal{R}, m \in \mathcal{M} \tag{5.3}$$

$$\sum_{i\in S}\sum_{j\notin S}\sum_{r\in\mathcal{R}}\sum_{m\in\mathcal{M}} x_{ijrm} \geq 1 \qquad \forall S \subseteq \mathcal{P}^{\mathrm{D}}, |S| \geq 2 \tag{5.4}$$

$$\sum_{i\in\mathcal{P}^{\mathrm{S}}}\sum_{j\in\mathcal{P}^{\mathrm{D}}}\sum_{m\in\mathcal{M}} x_{ijrm} \leq 1 \qquad \forall r \in \mathcal{R} \tag{5.5}$$

$$\sum_{i\in\mathcal{P}^{\mathrm{D}}}\sum_{j\in\mathcal{P}}\sum_{m\in\mathcal{M}} \frac{d_i x_{ijrm}}{q_m} = f_r \qquad \forall r \in \mathcal{R} \tag{5.6}$$

$$\sum_{(i,j)\in\mathcal{A}} \frac{f_r x_{ijrm} t_{ijm}}{365} \leq n_{rm} \qquad \forall r \in \mathcal{R}, m \in \mathcal{M} \tag{5.7}$$

$$\sum_{r\in\mathcal{R}} n_{rm} \leq \bar{n}_m \qquad \forall m \in \mathcal{M} \tag{5.8}$$

$$\sum_{j\in\mathcal{P}}\sum_{r\in\mathcal{R}}\sum_{m\in\mathcal{M}} \frac{d_i}{x_{ijrm} f_r}(1 + B) \leq y_i \qquad \forall i \in \mathcal{P}^{\mathrm{D}} \tag{5.9}$$

$$f_r \in \mathbb{R}^{\geq 0} \qquad \forall r \in \mathcal{R}, m \in \mathcal{M} \tag{5.10}$$

$$n_{rm} \in \mathbb{Z}^{+} \qquad \forall r \in \mathcal{R}, m \in \mathcal{M} \tag{5.11}$$

$$x_{ijrm} \in \{0, 1\} \qquad \forall (i, j) \in \mathcal{A}, r \in \mathcal{R}, m \in \mathcal{M} \tag{5.12}$$

$$y_i \in \mathbb{R}^{\geq 0} \qquad \forall i \in \mathcal{P}^{\mathrm{D}} \tag{5.13}$$

The objective function (5.1) minimizes total annual cost for both chartering and operating the LNG tankers and for building and running the LNG infrastructure at the ports of demand. Note that no particular specification of the cost function is used to emphasize that no restrictions are imposed on the type of the cost function (as e.g. linearity). Constraints (5.2) require each port of demand to be visited by exactly one rotation. The set of constraints (5.3)-(5.5) defines the tanker routes. Constraints (5.3) and (5.4) are flow conservation and subtour elimination constraints, respectively. Constraints (5.5) state that each route can leave a supply port at most once. Routes, for which the left-hand side of its corresponding constraint is zero, represent empty routes that are not used. The frequency f_r of each route is determined by constraints (5.6). The left-hand side equals the total demand served on route r divided by the tanker capacity used on that route. The integer number of LNG tankers required on each route is determined by constraints (5.7). The left-hand side represents the absolute number of days of operational time required to serve demand on route r with a single tanker, divided by the available number of days per year. An upper limit on the total number of tankers used per tanker type may be imposed by constraints (5.8). Constraints (5.9) determine the necessary onshore infrastructure capacity at each port i. It is required to be at least as large as the amount of LNG received at each tanker visit plus a relative buffer capacity B. Finally, constraints (5.10)-(5.13) define the domain of the decision variables.

In the above formulation the amount of LNG unloaded at a port depends on the route. It equals the tanker capacity times the port's share of demand compared to the demand of the other ports on the same route. Hence, the model allows for partially loaded LNG tankers. As discussed above we furthermore assume a steady demand for LNG over time and thus periodic LNG deliveries. The maximum amount of LNG on stock at a location thus never exceeds the amount of LNG unloaded at each delivery. Fig. 5.4 illustrates the stock level at a single port that is the only port on a route on which two LNG tankers operate.

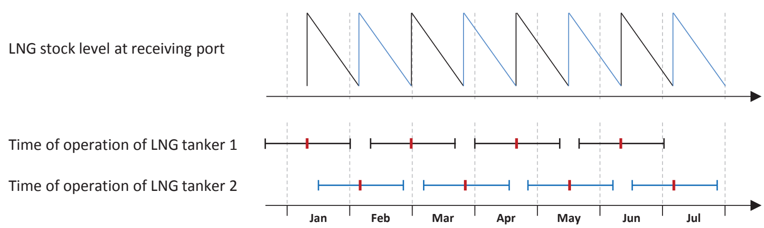


Figure 5.4: Illustration of the LNG stock level at a port that is supplied by a tanker service consisting of two tankers operating in parallel. The bars for each tanker illustrate their time of operation, with the red marks representing the time of delivery at the port.

The arc-based formulation generally is non-linear in its objective function and also in some of its constraints. Depending on the exact specification of its objective function, it can be linearized at the cost of a larger set and number of decision variables and constraints. The problem quickly grows to intractable size due to the number of decision variables and constraints that exponentially depend on the number of ports of demand. In the next section we thus present a path-based formulation that allows to easily incorporate non-linearities.

5.3.2 Path-based model formulation

In the arc-based formulation we can observe the following: once the routes and the corresponding tanker types are defined, i.e. once the x_{ijrm} variables are fixed, the values of the remaining decision variables follow directly from equations (5.6)-(5.9). Fixing the x_{ijrm} variables corresponds to defining a set of routes and the tanker type used on each of them. In other words, once a route and the tanker type used on that route are decided, the route frequency, the number of tankers and the infrastructure capacity at the ports along the route can be deduced. Let $\mathcal{S}$ denote the set of all route-tanker type combinations s. We will call a route-tanker type combination s with the associated route frequency f_s, number of LNG tankers n_s, LNG tanker capacity q_s and the port infrastructure capacities $(y_1, ..., y_{|\mathcal{P}^{\mathrm{D}}|})_s$ a *partial solution*. A partial solution can also be seen as a cluster of ports which is served by the same LNG tanker type. The cost of a partial solution is required to be independent of other partial solutions. With f_s, n_s, q_s and $(y_1, ..., y_{|\mathcal{P}^{\mathrm{D}}|})_s$ being parameters of a partial solution, we can hence calculate the corresponding cost of each partial solution.

Example: Let us consider a partial solution for the route Qatar-Singapore-Shanghai-Qatar (with Qatar being the port of supply) and a Q-max tanker type of 265 000m^3 capacity as an example. The demand in Shanghai equals 12 times the demand in Singapore and therefore on each service 1/13 of the tanker's capacity ($\approx$ 20 400m^3) is unloaded in Singapore and 12/13 ($\approx$ 244 600m^3) are unloaded in Shanghai on each tanker round trip. The required onshore storage capacity at each location is calculated as the absolute amount of LNG unloaded at each delivery plus some buffer capacity, i.e. the required infrastructure capacity is much smaller in Singapore than in Shanghai. If the total annual demand for LNG at both ports is 11 000 000m^3, approximately 41.5 tanker loads are required throughout the whole year. As a single service – starting in Qatar, delivering the LNG to Singapore and Shanghai and returning to Qatar – takes around 35 days, 4 tankers are required to do 41.5 round trips during a single year (365 days).

If all partial solutions were given, we could solve the problem by optimally combining these partial solutions such that each port is covered by exactly one partial solution. This translates into a set-partitioning model with a set $\mathcal{S}$ of partial solutions defined by a route-tanker type pair $s = (r, m)$ with $r \in \tilde{\mathcal{R}}$ and $m \in \mathcal{M}$, and a set $\mathcal{P}^{\mathrm{D}}$ of ports of demand. Note that we use $\tilde{\mathcal{R}}$ to denote the set of all possible routes, which should not be confused with the set $\mathcal{R}$ used in the arc-based formulation. Let $\hat{c}_s$ be the cost of partial solution $s \in \mathcal{S}$ and let a_{is} denote whether port of demand $i \in \mathcal{P}^{\mathrm{D}}$ is covered by partial solution s ($a_{is} = 1$) or not ($a_{is} = 0$). n_{ms} denotes the number of tankers of type m used on service s. The resulting set-partitioning model is

$$\min \quad \sum_{s \in \mathcal{S}} \hat{c}_s x_s \tag{5.14}$$

$$\text{s.t.} \quad \sum_{s \in \mathcal{S}} a_{is} x_s = 1 \qquad \forall i \in \mathcal{P}^{\mathrm{D}} \tag{5.15}$$

$$\sum_{s \in \mathcal{S}} n_{ms} x_s \leq \bar{n}_m \qquad \forall m \in \mathcal{M} \tag{5.16}$$

$$x_s \in \{0,1\} \quad \forall s \in \mathcal{S} \tag{5.17}$$

The partial solutions can either be enumerated in advance or, if the number of possible partial solutions is too large, the LP-relaxation of model (5.14)-(5.17) can be solved by generating partial solutions on the fly by using a delayed column generation algorithm. Note that almost any type of cost function can be used to define the cost $\hat{c}_s$ of a partial solution. The path-based formulation thus allows to include non-linearities in the cost while still solving a linear master problem. In Section 5.4.1 we provide a detailed description of the cost components used to define the cost of a partial solution $\hat{c}_s$ for the problem instance studied.

5.3.3 Enumeration and pre-selection of partial solutions

The number of partial solutions is of size $|\tilde{\mathcal{R}}| \times |\mathcal{M}|$. Since the size of set $\tilde{\mathcal{R}}$ depends exponentially on the number of ports of demand, $|\mathcal{P}^{\mathrm{D}}|$, and linearly on the number of ports of supply, $|\mathcal{P}^{\mathrm{S}}|$, so does the set of partial solutions $\mathcal{S}$. Based on the following observations, however, we can reduce the set $\mathcal{S}$ of partial solutions significantly: for each subset of ports of demand, different routes exist and different tanker types can be used on each of these routes. Each pair of a route and a tanker type represents a partial solution. For a given set of ports, there will be at least one route-tanker type pair that dominates the others in terms of cost. For each unique subset of ports we hence only need to include one partial solution, namely the one that dominates all other partial solutions that cover the same subset of ports of demand. If the number of tankers is limited for some tanker types and prohibitive (i.e. it would not allow to serve all routes with the same tanker type), we can still reduce the set by only keeping one partial solution per unique subset of ports and tanker type. By simple cost comparison we can thus eliminate a large part of the partial solutions in advance. The set of partial solutions that finally needs to be included in the set-partitioning model (5.14)-(5.17) could be reduced to the size $2^{|\mathcal{P}^{\mathrm{D}}|}-1$, if the number of tankers per tanker type is unlimited, and to $(2^{|\mathcal{P}^{\mathrm{D}}|}-1)|\mathcal{M}|$ if the number of tankers per tanker type is limited.

5.4 Case study

The presented modelling and solution approach have been applied to an industrial case study that was conducted in collaboration with a major liner shipping company. In the following sections the underlying data and the results are presented. Section 5.4.1 provides a detailed description of the cost functions as used in the path-based model formulation. In Section 5.4.2 we present selected scenarios that reflect different assumptions and allow to assess their impact.

In Section 5.4.3 results for the different scenarios are presented and discussed. As some of the input parameters are subject to large fluctuations over time, we present further results based on a sensitivity analysis in Section 5.4.4.

Eight locations along the route Asia-Europe (see Fig. 5.3) had been preselected by the company as potential future points of supply of LNG for their container vessel fleet as well as for spot market demand. The only loading port in the study is Qatar, the world's largest producer of LNG. We were further provided with LNG demand estimates for each of the unloading ports. The data is summarized in Table 5.1. The ports of Rotterdam and Shanghai represent the largest points of demand. Both ports are strategic hubs for container shipping – Rotterdam in Europe and Shanghai in Asia – and therefore the most frequented ones for container vessels. The major part of the demand at these two ports is dedicated to fueling container ships, but it may also be used for satisfying demand on the growing spot market. We note that the values in Table 5.1 represent expected future demand as estimated by the company and not the current demand for LNG. Even though LNG infrastructure may already exists at some of these ports, we assume, however, that all the infrastructure capacity required by the liner shipping company will have to be newly constructed at each of the ports for two reasons: first, the capacities at the ports with existing infrastructure are expected not to be sufficient to additionally handle the demands estimated by the company. The demand for LNG in Rotterdam estimated by the company, for example, would already make up around 85% of the terminal's capacity for small scale distribution when put into operation at the end of 2016 (Gate terminal, 2014). The second reason is that many existing LNG terminals simply do not provide an LNG bunkering option. Adding an LNG bunkering option is not straightforward, because many additional regulations regarding design safety and separation distances have to be satisfied. Furthermore, most of the existing LNG terminals are designed for low frequency, high volume transfers (DNV-GL, 2014).

Location	Demand [m^3]	Demand as factor of Malta demand
Rotterdam	4893.6K	46×
Algeciras	851.1K	8×
Malta	106.4K	1×
Port Said	212.8K	2×
Salalah	638.3K	6×
Jebel Ali	638.3K	6×
Singapore	851.1K	8×
Shanghai	10 212.8K	96×
Total	18 404.3K	173×

Table 5.1: Locations and corresponding LNG demand estimates

Due to the long planning horizon, the LNG tankers are expected to be newly built and we assume that LNG tankers of any capacity up to Q-Max size (265 000m^3), the largest LNG tankers currently in operation, are available in the case study. Hence, the discrete set of tankers, $\mathcal{M}$, used in the case study consists of around 260 different tankers. Approximating cost functions are used to estimate the charter rates and fuel consumption rates for each tanker. These and the other cost functions used in the case study are presented in the following subsection.

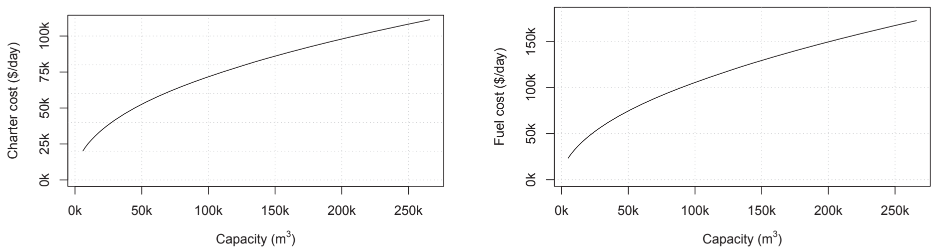
5.4.1 Cost functions

The annual cost $\hat{c}_s$ of a partial solution in the path-based formulation is composed of different cost factors (of which some have already been outlined Section 5.3.1). All cost components are presented as annual costs in thousand US dollars (k\$) if not stated otherwise.

The **charter cost** of a partial solution s, denoted by $\hat{c}_s^{\text{ch}}$, represents an annual charter cost for the number of LNG tankers of capacity q_s needed to serve annual demand on the corresponding route. As an estimate for the daily charter rate of a tanker with capacity q_s in thousand US dollars we use the function $9.0616 \cdot q_s^{0.4492}$ that is based on empirical data for tankers and their corresponding charter rates. The charter rate function as used in this case study is shown in Figure 5.5a. The annual charter cost of a partial solution $s \in \mathcal{S}$ depends on the tanker capacity q_s in thousand cubic meters and the number n_s of tankers needed on the route:

$$\hat{c}_s^{\text{ch}} = 365 \cdot (9.0616 \cdot q_s^{0.4492}) \cdot n_s \tag{5.18}$$

We would like to emphasize that in practice the charter rate is extremely volatile and dependent



(a) Charter cost function (b) Fuel cost function (HFO, sailing only)

Figure 5.5: Daily charter and fuel cost functions based on the tanker capacity

on current demand and supply. The used function is therefore at best an approximation to compare different sizes of tankers.

The **fuel cost** has been estimated in a similar fashion, based on empirical data for LNG tankers that run on heavy fuel oil. Note that although the new regulations regarding sulphur emissions will also apply to LNG tankers, our fuel cost estimates are based on HFO fueled LNG tankers as an approximation due to the lack of available data. We will, however, take possible differences in fuel cost under the new regulation into account by defining different cost scenarios. We distinguish between the consumption of HFO, which is used to fuel the tankers while sailing, and MGO, which is used during port stays while loading or unloading LNG. The respective fuel cost functions for partial solution $s \in \mathcal{S}$ are defined as

$$\hat{c}_s^{\mathrm{HFO}} = (10.4293 \cdot q_s^{0.5028}) \cdot t_s^{sail} \cdot p^{\mathrm{HFO}} \tag{5.19}$$

$$\hat{c}_s^{\mathrm{MGO}} = (2.9500 \cdot q_s^{0.5652}) \cdot t_s^{\mathrm{port}} \cdot p^{\mathrm{MGO}} \tag{5.20}$$

The fuel cost $\hat{c}_s^{\mathrm{HFO}}$ equals the tanker type dependent fuel consumption per day multiplied with the accumulated annual sailing time of this partial solution, t_s^{sail}, and is further multiplied with the fuel price per ton, p^{HFO}. The parameter t_s^{sail} is obtained through multiplication of the duration of a single route times the frequency of the route, f_s. The daily fuel consumption while sailing is based on an assumed speed of 18 knots. Figure 5.5b shows the daily fuel cost depending on the tanker capacity. The cost function $\hat{c}_s^{\mathrm{MGO}}$ for burning MGO during port stays represents the tanker dependent in-port fuel consumption per hour multiplied with the total in-port time, t_s^{port}, and multiplied with the fuel price per ton, p^{MGO}. Bunker prices of \$300 and \$450 for HFO and MGO, respectively, are assumed in the base case of the study. The total fuel cost of a partial solution s hence is $\hat{c}_s^{\mathrm{fuel}} = \hat{c}_s^{\mathrm{HFO}} + \hat{c}_s^{\mathrm{MGO}}$. In practice the fuel cost function may further depend on the ship type, the ship's load and many other factors (see e.g. Psaraftis and Kontovas, 2014, for a discussion of fuel cost estimation). The path-based modelling framework allows to incorporate basically any type of fuel cost function, if the necessary information is available.

The annual **infrastructure cost** consists of capital expenditures (CAPEX) and operational expenditures (OPEX). We will consider the same, capacity-dependent investment cost at all ports. Even thoug investment options and cost may differ between different ports, the same cost function is used for all ports due to the lack of reliable data for each individual port. In order to estimate the capital expenditures for LNG terminals of different capacities, we use a capacity factor based estimator (see e.g. Westney, 1997). It uses historical records to estimate the cost of similar projects. The formula as shown below further assumes a non-linear increase in cost with increasing capacity, reflecting economies of scale (Fig. 5.6). The parameters $\bar{y}_1, \bar{y}_2$ and $\mathrm{CAPEX}(\bar{y}_1), \mathrm{CAPEX}(\bar{y}_2)$ represent capacities and respective capital expenditures of reference projects:

$$\frac{\mathrm{CAPEX}(\bar{y}_2)}{\mathrm{CAPEX}(\bar{y}_1)} = \left(\frac{\bar{y}_2}{\bar{y}_1}\right)^{\delta} \tag{5.21}$$

The parameter δ, also called *capacity factor*, is estimated based on two anchor points provided by the company. They correspond to two existing LNG terminals of capacities $28\,500\mathrm{m}^3$ and $114\,000\mathrm{m}^3$, and CAPEX of \$94M and \$164M, respectively. The resulting capacity factor used in our study is $\delta = 0.4015$. The CAPEX of any LNG terminal with capacity y can thus be calculated using the formula:

$$\mathrm{CAPEX}(y) = \mathrm{CAPEX}(\bar{y}_1) \cdot \left(\frac{y}{\bar{y}_1}\right)^{0.4015} \tag{5.22}$$

Equivalent to the arc-based model formulation, we require the capacity of an LNG terminal at port i to be as large as the amount of LNG that is received with each tanker visit plus a buffer. For a given partial solution, the amount delivered at a port corresponds to its share of demand on the route multiplied with the capacity of the tanker used. Hence, the storage capacity y_{is} of a port i of a partial solution s can be determined as

$$y_{is} = \frac{a_{is} d_i}{\sum_{i \in \mathcal{P}} a_{is} d_i} q_s (1 + B) \tag{5.23}$$

The annual OPEX of an LNG terminal are estimated to lie at around 5% of the CAPEX. Let t^{life} be the expected lifetime of the terminal. The total annual infrastructure cost of a partial solution is then calculated as the sum of the capital expenditures per year and the annual operational expenditures, i.e.

$$\hat{c}_s^{\text{inf}} = \sum_{i \in \mathcal{P}} (\mathrm{CAPEX}(y_{is})/t^{\text{life}} + 0.05 \cdot \mathrm{CAPEX}(y_{is})) \tag{5.24}$$

In the study we assume a lifetime of 30 years for the LNG terminals.

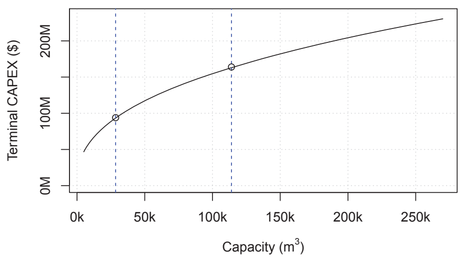


Figure 5.6: Terminal capital expenditures depending on the storage capacity. The two reference points used to calculate the capacity factor exponent δ are highlighted.

The **port call cost** is the cost charged by a port each time an LNG carrier berths and it is assumed to be equal at all ports considered. The cost factor $\hat{c}_s^{\text{port}}(q_s)$ represents the annual port call cost of a partial solution and depends on the carrier capacity q_s, the number of ports that are covered by the corresponding partial solution, $|\mathcal{P}_s| = \sum_{i \in \mathcal{P}} a_{is}$, and the frequency of the service, f_s. The cost per visit depends on the tanker's capacity, which is divided into three capacity classes and corresponding port call fees. The port call cost of a solution in thousand

US dollars is defined as

$$\hat{c}_s^{\text{port}} = \begin{cases} 60 \cdot |\mathcal{P}_s| \cdot f_s & \text{if } q_s < 50 \\ 150 \cdot |\mathcal{P}_s| \cdot f_s & \text{if } 50 \leq q_s < 120 \\ 300 \cdot |\mathcal{P}_s| \cdot f_s & \text{if } 120 \leq q_s \end{cases} \tag{5.25}$$

Canal transit costs occur whenever a ship uses a canal. In our study the Suez canal is the only relevant canal. We use a function that linearly depends on the capacity of the tanker in order to approximate a single transit through the canal. For the smallest tanker considered (5000m^3) we assume a cost of \$100k per transit and for the largest carrier considered (265 000m^3) a cost of \$500k per transit. Canal transit costs for tankers with a capacity in between are obtained through linear interpolation. The annual canal transit costs of a partial solution that uses the Suez canal are thus calculated as

$$\hat{c}_s^{\text{canal}} = 2 \cdot f_s \cdot \left(100 + \frac{q_s - 5}{265 - 5} \cdot (500 - 100)\right) \tag{5.26}$$

We further consider **inventory cost** for LNG on board the tankers during transport and at the destination ports before consumption. With a partial solution given, we can calculate the average amount of LNG on tankers and in onshore storage tanks throughout a year. Let $\bar{l}_s$ denote the average inventory of LNG in thousand cubic meters of partial solution s throughout a year. Assuming a value of \$220/m^3 of LNG ($\approx$ \$9.16/mmBTU of LNG) and an inventory cost rate of 6% per year in our study, the annual inventory cost of a partial solution in thousand US dollars is defined by

$$\hat{c}_s^{\text{inv}} = \bar{l}_s \cdot 220.00 \cdot 1.06 \tag{5.27}$$

Any of these cost functions can be adapted to different settings. Although we neglect the loss of LNG due to boil-off in our study, it can also be accounted for in path-based formulations (see e.g. Grønhaug et al., 2010). Finally, the total cost of a partial solution is given by the sum of all cost factors,

$$\hat{c}_s = \hat{c}_s^{\text{ch}} + \hat{c}_s^{\text{fuel}} + \hat{c}_s^{\text{inf}} + \hat{c}_s^{\text{port}} + \hat{c}_s^{\text{canal}} + \hat{c}_s^{\text{inv}} \tag{5.28}$$

We can calculate the total cost of any partial solution using equations (5.18)-(5.28) and, as described above in Section 5.3.3, eliminate many partial solutions by simple cost comparison, as for each subset of ports one partial solution will dominate all the other partial solutions for that same subset of ports.

Figure 5.7 illustrates costs of different partial solutions for the route *Qatar–Jebel Ali–Salalah–Qatar*, with each of these partial solutions corresponding to a different tanker capacity in the range 5000-265 000m^3. The partial solution with the lowest cost is denoted by the dashed line. All costs represent annual costs for satisfying the demand at the two locations. The charter cost

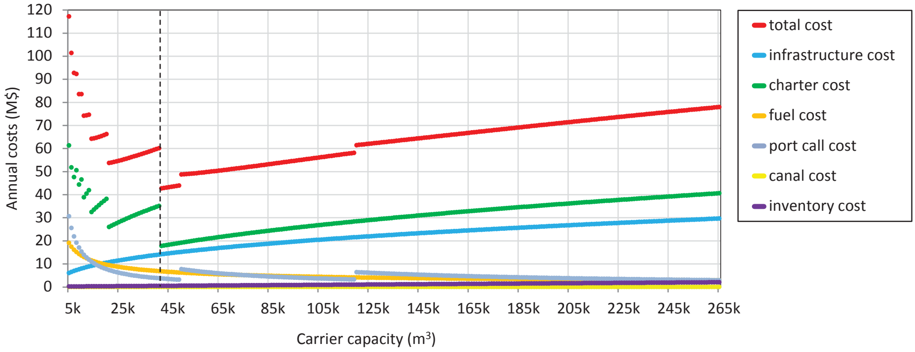


Figure 5.7: Costs of different partial solutions for the route Qatar–Jebel Ali–Salalah–Qatar. For the given route, the tanker capacity of 42 000m^3 dominates all other tanker capacities in terms of total cost.

is increasing with the tanker capacity and the steps result from different numbers of tankers needed to satisfy demand. For tanker capacities of 21 000-41 000m^3, for example, two tankers are necessary to satisfy demand at the two locations. Two tankers of 21 000m^3 would have a much higher utilization than two tankers of 41 000m^3, though. With a tanker capacity of 42 000m^3 and higher, only a single tanker is necessary to satisfy annual demand. The fuel cost is decreasing with increasing tanker capacities, as larger tankers are more fuel efficient given the same total amount of LNG being transported (see Figure 5.5b). The port call costs are different for the three capacity classes of tankers (see the port call cost function (5.25)), and within each class decreasing with increasing tanker capacities, as larger capacities imply a lower number of deliveries of LNG. Note that for the location subset {Jebel Ali, Salalah} one alternative route exists (*Qatar–Salalah–Jebel Ali–Qatar*) whose corresponding partial solutions are not represented in this graph. Only the lowest-cost route-tanker type combination of the location subset {Jebel Ali, Salalah} would finally be included in the set-partitioning problem.

5.4.2 Scenarios

The cost functions presented above are just one way of defining different types of cost and associated assumptions. We will refer to the above described scenario as **scenario A**. The simple structure of the path-based problem formulation allows for any type of cost functions that depend on the route and tanker type of a partial solution. It further allows to model different underlying assumptions. We thus present two other scenarios that differ from scenario *A*. The two scenarios are not exhaustive, of course, but represent examples of changing underlying decisions and corresponding assumptions and how they can be evaluated using the model.

Scenario B In scenario A, the onshore storage capacities are chosen such that they exactly meet the requirements imposed by the chosen service (as defined by equation (5.23)). That is, the capacities are required to satisfy just the amount of LNG that is unloaded at each tanker visit, plus some buffer capacity. The assumption makes the results highly reliant on the simplified routing model that is part of the overall model and may be very restrictive for finding good inventory routing solutions at an operational level once the infrastructure and tanker fleet are fixed. In scenario B we require the onshore capacity to be at least as large as the capacity of the tanker type that is assigned to the corresponding port (plus some buffer), independent of the amount unloaded. The modified assumption is implemented by replacing equation (5.23) of scenario A by expression

$$y_{is} = a_{is} q_s (1 + B) \tag{5.29}$$

A partial solution can be considered as a clustering of ports, where each cluster of ports is served by the same LNG tanker type. In any solution under scenario B the infrastructure capacity of a port will be at least as large as a full load of the LNG tanker type that is assigned to this port. This allows for a much wider range of inventory routing solutions at an operational level, including routings that are not periodic or routings that require a tanker to always unload its full load. Scenario B is interesting from an industrial point of view because the majority of LNG tankers currently under construction are of the membrane type. These are known to suffer from sloshing and consequently rather strict filling levels apply to these tankers (Delorme et al., 2005), possibly discarding the simplified routing solutions determined by the model.

Scenario C In a third scenario, scenario C, we assume that tanker charter costs only apply for the actual time of utilization, not for the whole year. Hence, no cost is assumed for tankers while not being used. Under scenario A and B idle times of LNG tankers are actually penalized, as charter costs are independent of utilization. Under scenario C the original charter cost function (5.18) is replaced by the modified charter cost function

$$\hat{c}_s^{\text{ch}} = 365 \cdot u_s \cdot (9.0616 \cdot q_s^{0.4492}) \cdot n_s \tag{5.30}$$

with u_s denoting the tanker utilization of the corresponding partial solution s. In this scenario the obtained charter cost can be interpreted as a lower bound on the expected charter cost. The scenario simulates a situation in which LNG tankers could be chartered just for the time they are utilized, i.e. idle times are not penalized.

5.4.3 Results

We implemented an algorithm for efficiently enumerating all partial solutions, which constitute the input to the set partitioning model. The set-partitioning models were generated in the OPL Script language and solved using CPLEX. The tests were executed on a machine with an

(a) Scenario A (b) Scenario B

Figure 5.8: Optimal clustering of ports. All ports within a cluster are served on round trips and by the same tanker type.

Intel Core i7-4600U CPU with 2.1GHz. The generation of the partial solutions takes around 30 seconds on average and the solution time of the set-partitioning model is less than a second.

The results for scenario A are summarized in Table 5.2. In the optimal solution all ports are clustered pairwise, as illustrated in Figure 5.8a. The charter costs dominate with almost making up half of the total costs, followed by fuel costs (21%) and infrastructure costs (14%). The share of the different cost factors, however, differs substantially between the various partial solutions. The partial solutions covering the ports of Shanghai and Rotterdam, which represent more than 80% of the total demand for LNG, also constitute more than 85% of the total cost. Due to Shanghai's and Rotterdam's large distances from Qatar, charter and fuel cost per transported unit of LNG are very high compared to the other cost components (50% and 22%, resp.). In the partial solution that covers demand in Malta and Port Said, the share of charter and fuel cost (36% and 14%, resp.) is significantly lower, while infrastructure cost and also canal transit cost have a much larger weight (30% and 13%, resp.). We further observe that for each cluster of ports the optimal tanker capacity corresponds to the capacity that minimizes the necessary number of LNG tankers but maximizes their utilization. The optimal infrastructure capacity per port is given in Table 5.5.

There are several trade-offs inherent in any solution. Clustering ports and assigning them to

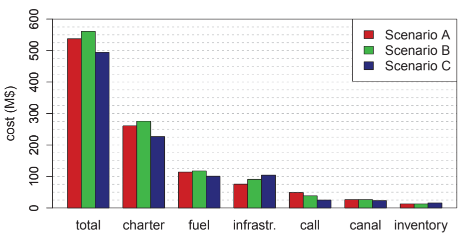


Figure 5.9: Comparison of cost of the optimal solution between the different scenarios.

a single tanker round trip allows to use larger, more cost-efficient tankers while keeping their utilization high. Scenario A further allows to reduce the cost for infrastructure, as only a fraction of the tanker load will be unloaded at each port visit. On the other hand, with more combined ports on a route the total sailing distance and the total number of port visits and canal transits (when required) are larger compared to serving each port individually, given the same tanker capacity.

	Characteristics				Annual cost (M$)						
Route	tanker cap. (m^3)	num. of tankers	annual frequency	utilization	charter cost	fuel cost	infrastr. cost	port call cost	canal cost	inventory cost	total cost
Algeciras - Rotterdam	255k	2	22.5	100%	79.7	35.6	27.0	13.5	21.8	5.0	182.7
Port Said - Malta	18k	1	17.7	99%	12.1	4.6	9.9	2.1	4.3	0.2	33.3
Jebel Ali - Salalah	42k	1	30.4	98%	17.7	6.7	14.1	3.6	0.0	0.4	42.7
Singapore - Shanghai	226k	4	49.0	100%	151.0	67.0	24.3	29.4	0.0	7.2	278.9
Total					260.6	113.9	75.4	48.7	26.1	12.8	537.5
Total (%)					48%	21%	14%	9%	5%	2%	100%

Table 5.2: Optimal solution to scenario A: Each row represents an optimal partial solution and its characteristics and costs. The first column indicates the route that is sailed in the corresponding partial solution, always starting and ending in Qatar. The following two columns denote the number and capacity of tankers used. The frequency denotes how often the round trip is sailed every year and equivalent to the number of port calls at each port along the respective route. The utilization indicates the relative time per year the tankers are in operation. The remaining columns denote the different annual cost per partial solution.

Table 5.3 summarizes the results obtained for scenario B, where the infrastructure capacity is required to be as large as the capacity of the tanker that serves the corresponding port, independent of the amount of LNG being unloaded. Scenario B is more restrictive than scenario A and the total annual cost of its optimal solution is 4.4% higher than the cost of the corresponding optimal solution in scenario A. As could be expected, the total infrastructure capacity is significantly higher than in scenario A (see Table 5.5). The effect of the assumption introduced under scenario B, however, is dampened by the use of smaller tankers and a larger number of individually served ports compared to the optimal solution of scenario A. The ports of Shanghai, Singapore, Salalah and Jebel Ali are served on individual round trips whereas they are served pairwise in scenario A (see Figure 5.8b). Scenario B's additional requirement represents a reduced incentive to combine multiple ports on a round trip and a greater incentive to use LNG tankers of smaller capacity. Again, in almost all cases the capacities of the tankers are chosen such that the number of tankers is minimized and their utilization is maximized. Overall, the optimal solution under scenario B is \$23.6M more expensive than the optimal solution under scenario A. In other words, if the problem of sloshing can be resolved and partial tanker loadings are feasible in the future, it may allow to reduce the total annual cost by more than 4%.

	Characteristics				Annual cost (M$)						
Route	tanker cap. (m^3)	num. of tankers	annual frequency	utilization	charter cost	fuel cost	infrastr. cost	port call cost	canal cost	inventory cost	total cost
Algeciras - Rotterdam	255k	2	22.5	100%	79.7	35.6	38.5	13.5	21.8	5.0	194.2
Port Said - Malta	18k	1	17.7	99%	12.1	4.6	13.3	2.1	4.3	0.2	36.6
Salalah	20k	1	31.9	98%	12.7	4.7	6.9	1.9	0.0	0.3	26.5
Jebel Ali	8k	1	79.8	53%	8.4	1.2	4.8	4.8	0.0	0.1	19.2
Singapore	43k	1	19.8	99%	17.9	7.2	9.4	1.2	0.0	0.6	36.3
Shanghai	206k	4	49.6	100%	144.9	64.2	17.7	14.9	0.0	6.7	248.2
Total					275.7	117.5	90.6	38.4	26.1	12.7	561.1
Total (%)					49%	21%	16%	7%	5%	2%	100%

Table 5.3: Optimal solution to scenario B: Each row represents an optimal partial solution and its characteristics and costs.

In scenario C charter costs are only incurred for the time tankers are in operation and not for the whole year, i.e. idle times are free of charge. A summary of the optimal solution for scenario C is given in Table 5.4. Port Said, Malta and Algeciras are served in a single LNG tanker round trip, while all other ports are served by individual round trips. The optimal solution appears to be in line with intuition: the average tanker size is significantly larger compared to scenario A. The main reason is that the larger the tanker is, the lower are the charter and fuel costs per unit of LNG transported. Larger tankers may, however, not be fully utilized throughout a year. While idle times of tankers incur a cost in Scenario A, they do not incur any cost in Scenario C, which explains why larger tankers are more economic under scenario C. Due to the increased tanker capacities, the onshore infrastructure capacities are even larger than under scenario B (see Table 5.5). The more impacting charter costs and fuel costs are reduced at the price of a less heavy increase in annual infrastructure cost. Note that in scenario C the required infrastructure capacity at a port is, like in scenario A, a function of the amount unloaded, which represents an incentive to combine ports on round trips.

Depending on which scenario is considered, the optimal strategic decisions can be very different, as e.g. the optimal onshore storage tank capacities indicate (Table 5.5). The relative difference in cost between the corresponding optimal solutions, however, is still rather moderate (Fig. 5.9), especially against the background of considerable uncertainty in the input parameters. The latter will be addressed in the following section through a sensitivity analysis.

5.4.4 Sensitivity Analysis

Many of the input parameters are subject to heavy fluctuations over time and therefore extremely hard to predict. By early 2016, the heavy fuel oil price has, against the forecasts, plummeted

Route	Characteristics: tanker cap. (m^3)	num. of tankers	annual frequency	utilization	Annual cost in million USD: charter cost	fuel cost	infrastr. cost	port call cost	canal cost	inventory cost	total cost
Rotterdam	265k	2	18.5	81%	65.4	29.5	19.6	5.5	18.5	4.6	143.1
Port Said - Malta - Algeciras	242k	1	4.8	35%	13.6	6.0	33.3	4.4	4.5	2.2	63.9
Salalah	49k	1	13.0	41%	7.7	3.1	9.9	0.8	0.0	0.5	21.9
Jebel Ali	24k	1	26.6	18%	2.5	0.7	7.5	1.6	0.0	0.2	12.4
Singapore	119k	1	7.2	36%	10.3	4.4	14.2	1.1	0.0	1.1	31.1
Shanghai	265k	4	38.5	78%	126.7	57.0	19.6	11.6	0.0	7.1	221.9
Total					226.2	100.6	104.0	24.9	23.0	15.6	494.3
Total (%)					46%	20%	21%	5%	5%	3%	100%

Table 5.4: Optimal solution to scenario C: Each row represents an optimal partial solution and its characteristics and costs.

Port	Scenario A	Scenario B	Scenario C
Rotterdam	228.1	267.8	278.3
Algeciras	39.7	267.8	184.8
Malta	6.3	18.9	23.1
Port Said	12.6	18.9	46.2
Salalah	22.1	21	51.5
Jebel Ali	22.1	8.4	25.2
Singapore	18.2	45.2	125.0
Shanghai	218.1	216.3	278.3
Total	567.0	864.2	1012.2
Total vs. Scenario A		+52.4%	+78.5%

Table 5.5: Optimal onshore infrastructure capacities in thousand m^3 for all ports and under different scenarios.

to a value below \$300 per tonne since August 2014, after having been above \$600 for more than three years. Similarly, charter rates may fluctuate heavily over time. The average daily charter rate for 12 months time charters for a 160 000m^3 LNG tanker, for example, has varied between \$30 000 and \$150 000 since 2010 (Platou, 2015). For other input parameters as e.g. the infrastructure cost (as a function of capacity) even current estimates are generally provided as interval estimates due to many factors of uncertainty. In this section we show how the presented modelling approach can be used to test the robustness of the above solutions against fluctuations of the fuel price, charter rates and expected infrastructure costs compared to the base case. The sensitivity analysis was conducted for scenarios A and B exclusively due to their practical relevance. The focus of the sensitivity analysis lies on the strategic infrastructure sizing and tanker investment decisions, as the reversion or adjustment of strategic decisions is usually not possible without considerable extra costs.

We tested seven different charter rate levels including the base case. As the charter rate is a function of the tanker capacity, the changes in charter costs are provided as relative deviations from the base cost function in percent. Values in the range −50% to +100% of the base charter cost function have been evaluated, i.e. the lowest representing half and the highest representing twice the base case charter rate. To evaluate the solution robustness against fluctuations in the fuel price we tested five different equally distant values in the range of \$150-\$750 per tonne of HFO, with \$300 corresponding to the base case. As in the base case, the price of MGO was generally set to 1.5 times the price of HFO. We furthermore evaluated the impact of infrastructure cost changes in the range of −50% to +100% of the annual infrastructure cost function (CAPEX+OPEX) as defined in the base case. Note that each parameter variation was tested individually against the base case of the corresponding scenario.

The results of the sensitivity analysis are summarized in Figures 5.10 and 5.11. Figure 5.10 shows selected properties of each case's optimal solution and how it changes with varying input parameters. In the given study we are mainly interested in how robust irreversible decisions like the number and size of tankers and the size of the onshore infrastructure are. Generally, the impact of all tested parameter fluctuations on the average optimal onshore storage capacities is very limited under both scenarios, with the average (optimal) infrastructure capacity varying between 71 000m^3 and 78 000m^3. The largest observed relative change in capacity for a single port was 17%. The optimal number of tankers and their capacity appears to be more sensitive to the parameter fluctuations, at least under scenario B. Under some parameter fluctuations the average number of tankers can vary by up to 33% and the average tanker capacity by up to 29% compared to the optimal solution of the base case.

Although the optimal solutions may look substantially different under some parameter realizations, the total cost savings compared to the optimal solution of the base case turn out to be relatively low. Figure 5.11 illustrates the relative changes in all cost components of an optimal solution compared to the base case. Figure 5.11e, for example, illustrates a case where the optimal solution of the base case remains optimal under all tested infrastructure cost variations in scenario A. This is not the case under scenario B, but Figure 5.11f shows that the adjustments of the optimal solution reduce the impact on some cost factors and increase the impact on others. Figure 5.12 shows the savings in total cost that could be achieved if reoptimization was possible after different parameter realizations. Interestingly it turns out that none of the reoptimized solutions reduces the total cost by more than 0.8% compared to the solution that is optimal for the base case. On the contrary, the solution to the base case remains optimal or very close to optimal under most of the tested parameter variations.

These results can be interpreted in two ways: the optimal solution to the base case appears to be quite robust to changes in the main cost functions, i.e. the optimal solution to the base case remains a very good solution under the evaluated parameter changes. On the other hand, however, it means that in practice there is very little leeway for the company to absorb cost increases in the investigated case study, even if major strategic decisions were reversible or adjustable in the future. These insights are very valuable, especially when it comes to risk assessment of large projects like the one considered.

5.5 Conclusion and Discussion

The paper has introduced a novel, non-linear model for solving a strategic infrastructure and tanker fleet sizing problem. The model decides the dimensions of a number of LNG depots that are meant to serve periodic large-scale demand for LNG but possibly also demand for LNG in the spot market. It further determines the types and number of LNG tankers that are necessary to transport LNG from loading ports to the depots at the destination ports. The model includes a simple routing component in order to analyze the trade-off between long-term strategic and operational costs. The model was developed as part of a study (conducted by one of the major global container lines) that examined the feasibility of operating container ships by LNG. The non-linear model proves difficult to solve (Boleda Molas, 2015) and consequently an equivalent set partitioning formulation and necessary preprocessing techniques are proposed. The proposed model allows to solve realistic problem instances that have been presented in the paper within a very short time. The presented modelling framework is highly flexible and allows to incorporate almost any type of cost functions and underlying assumptions. Quick solution times allow decision makers to investigate different scenarios and better understand cause-and-effect relationships. The results for the presented case study show that the costs of transporting the LNG from its source to its destinations vastly exceed the cost of building and operating the necessary onshore infrastructure when considering a 30 year life time of the infrastructure. Hence, operations should drive infrastructure decisions and not vice versa, as traditionally done. These results emphasize the necessity and advantages of optimizing infrastructure investment decisions and interdependent operational decisions simultaneously. Due to the volatility of the input parameters as well as due to the long planning horizon of the study, any optimal solution is associated with a large degree of uncertainty. Consequently a sensitivity analysis has been conducted. The results show that the obtained solutions are quite robust in terms of solution quality, but that any changes in individual cost factors will be almost fully reflected in the total cost.

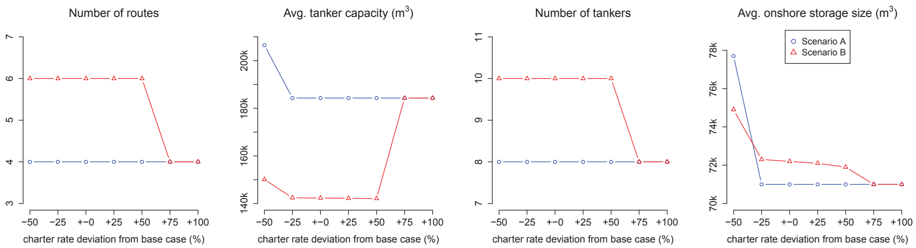
It is worth noting that the study was initiated in a time when oil prices were much higher compared to what they are at the time of writing. At this moment of time, however, it is not clear yet how the stricter sulphur regulations will affect marine gas oil and marine diesel oil prices once they become globally binding in 2020. Due to the large space requirements of engines and fuel tanks, container vessels that can run on both LNG and diesel oil are not a realistic option so far. Operating a mixed fleet, however, represents a feasible solution for becoming less dependent on either diesel oil or LNG prices. Another viable option are LNG-ready container vessels that can be retro-fitted for LNG fuel at a later stage; the first of these LNG-ready vessels have been delivered to United Arab Shipping Company recently (United Arab Shipping Company, 2014). Generally, LNG's competitiveness against diesel oil is expected to increase due to a quickly growing spot market that results in many new LNG importing and exporting countries (Wood, 2012).

The presented solution approach relies on a single-period model, assuming a stable context over the planning horizon. In practice, the shipping market has shown to be rather unstable in the long term; on the other hand, the realization of future demand is generally not known at the time of taking strategic decisions because of the dynamic nature of the shipping market. Possibly for that reason most of the literature addressing fleet size and mix problems considers single-period problems assuming deterministic demand (Pantuso et al., 2014). A possible extension of the presented problem is one towards a multi-period problem, similar to maritime fleet renewal problems. The presented model, however, allows re-optimization at any given time based on an existing infrastructure and tanker fleet. Allowing to charter out or lay up tankers and similarly to add tankers to an existing fleet or to extend the infrastructure can be implemented in the presented model in a straightforward way by adding corresponding sets of existing tankers and simple modifications of the cost functions.

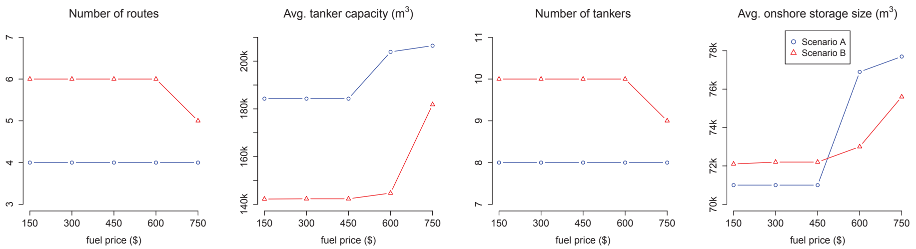
Several other interesting extensions of the problem studied can be considered in future work. The large degree of stochasticity in the input data suggests the use of Stochastic Programming or Robust Optimization to deal with the uncertainty. This includes long-term demand cycles as they occur in the shipping industry (see Stopford, 2009, e.g.). A stochastic model could optimize over a larger set of scenarios and select solutions that are robust to fluctuations. Since the deterministic model can be solved within short solution times, it seems realistic, from a computational point of view, to make the move towards a stochastic model. Obtaining reasonable probability distributions for all input parameters represents a major challenge, though. Another possible extension is to broaden the decision problem and include the selection of ports that shall accommodate LNG terminals. This change would require to explicitly include the container ship routes in the model, since fewer refueling options may imply that the container ships need larger LNG fuel tanks. There is a tradeoff between the LNG tank size on board of container ships and the vessels' container loading capacity. Thus, such a change would make the problem much more complex.

5.6 Acknowledgements

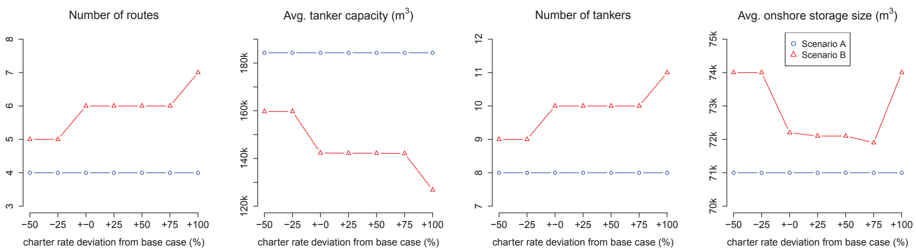
The research was supported by The Danish Strategical Research Council and The Danish Energy Technology Development and Demonstration Program (EUDP) under the ENERPLAN and GREENSHIP project.



(a) Solution properties for scenarios A and B under varying charter rates. The x-axis shows the % deviation from the scenario's base charter rate function.

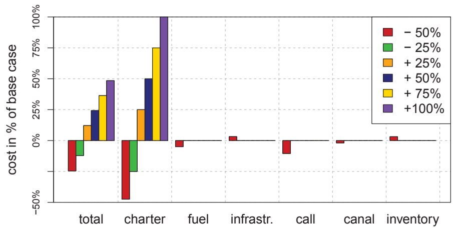


(b) Solution properties for scenarios A and B under varying fuel prices. The x-axis shows the different tested fuel prices.

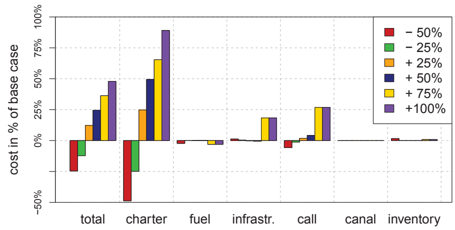


(c) Solution properties for scenarios A and B under varying infrastructure costs. The x-axis shows the % deviation from the scenario's base infrastructure cost function.

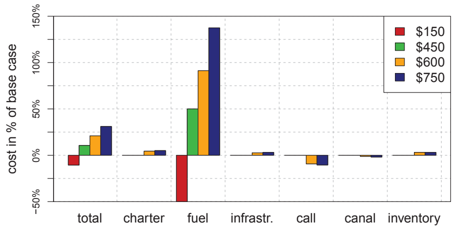
Figure 5.10: Properties of the optimal solution for different scenarios and parameters



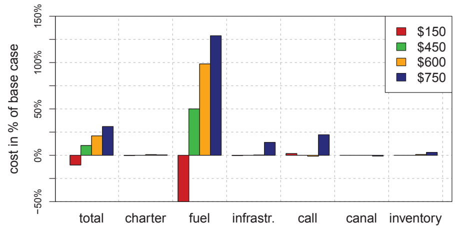
(a) Scenario A: Impact of fluctuations in the charter rates on the different cost factors



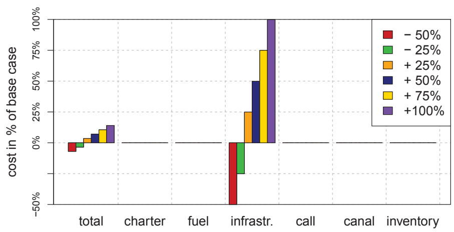
(b) Scenario B: Impact of fluctuations in the charter rates on the different cost factors



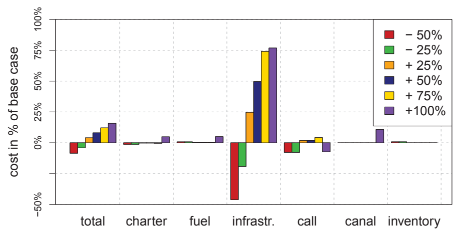
(c) Scenario A: Impact of fluctuations in the fuel price (base: $300) on the different cost factors



(d) Scenario B: Impact of fluctuations in the fuel price (base: $300) on the different cost factors

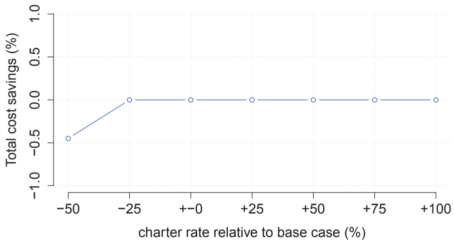


(e) Scenario A: Impact of fluctuations in the infrastructure cost on the different cost factors

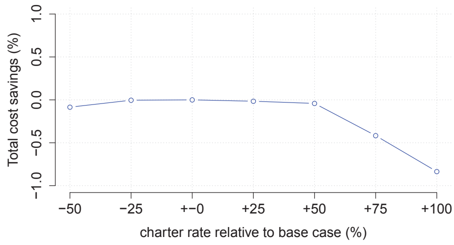


(f) Scenario B: Impact of fluctuations in the infrastructure cost on the different cost factors

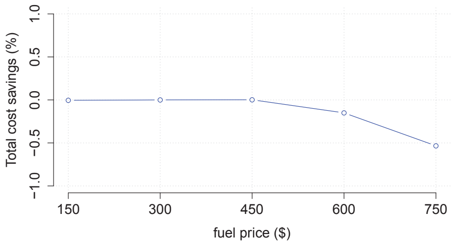
Figure 5.11: Impact of varying input parameter values on cost factors of optimal solutions under scenario A and B. Each subfigure shows the relative change in different annual cost factors (total costs, charter costs, fuel costs, infrastructure costs, port call costs, canal transit costs, inventory costs) given the change of a selected input parameter (e.g. charter rate). All values represent relative changes compared to the costs in the optimal solution of the corresponding scenario's base case.



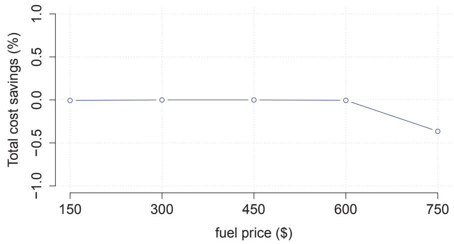
(a) Scenario A, varying charter rates



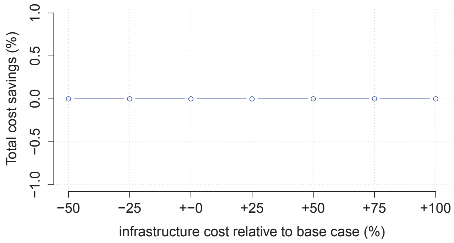
(b) Scenario B, varying charter rates



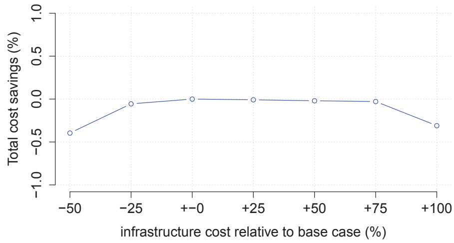
(c) Scenario A, different fuel prices



(d) Scenario B, different fuel prices



(e) Scenario A, varying infrastructure cost



(f) Scenario B, varying infrastructure cost

Figure 5.12: Potential cost savings obtained through reoptimization after parameter changes compared to the optimal solution of the base case.

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The modern economy relies on cheap and reliable transportation of goods all around the globe. Liner shipping networks represent an important link in the global supply chain, as they connect countries and continents over long distances at low transportation costs and with a comparatively low environmental impact.

In this thesis we present Operations Research tools that help decision makers in the liner shipping industry to find solutions to complex decision problems.
The developed models and methods address strategic, tactical and operational challenges. These include the design of liner shipping networks, the scheduling of services and the routing of cargo - all under consideration of practical restrictions and customer requirements.

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